Answers to basic exercises

1. **What is the mass of 1 dm^3 of mercury?**
   The volume is: $V = 10^{-3}\ m^3$
   The relative density is: 13.5. The density of mercury in kg/m^3 is the product of its relative density and the density of water in kg/m^3: $(13.5) \times (10^3) = 13'500\ kg/m^3$
   From Eq. 1-1, the mass is given by : $m = \rho \cdot V = 13'500 \times 10^{-3} = 13.5\ kg$

2. **What is the density of sea water at 5°C knowing that the average salinity is 35 grams per litre?**
   The volume of 1 litre of seawater is: $V = 1 \times 10^{-3}\ m^3$
   The mass of 1 litre of seawater is 1 kg plus 0.035 kg : m = 1.035 kg
   The density from Eq. 1-1 is $\rho = m / V = 1.035 / 10^{-3} = 1.035 \times 10^3\ kg/m^3$
   The addition of salt has a similar influence on density as a decrease in temperature: salty water at 20°C will have the same density as pure water at 5°C.

3. **What is the density of air at a pressure of $1 \cdot 10^5$ Pa and a temperature of 30°C?**
   The pressure is $P = 10^5\ Pa$
   The gas constant for dry air is $R = 29.3\ m/K$
   The absolute temperature is about $T = 273 + 30 = 303\ K$
   Standard gravity is $g = 9.81\ m/s^2$
   The density of air is given by Eq. 1-2:
   $\rho = 10^5 / (29.3 \times 303 \times 9.81) = 1.15\ kg/m^3$
   Thus one litre air is about one thousand times lighter than a litre of water.

4. **In a closed air tank we have a pressure of $10 \cdot 10^5$ Pa at 0°C. As the maximum pressure authorised for the vessel is $12 \cdot 10^5$ Pa, what is the maximum temperature admissible?**
   The pressure at 0°C is: $P_0 = 10 \times 10^5\ Pa$
   The ma· pressure is: $P_{ma\cdot} = 12 \times 10^5\ Pa$
   The absolute temperature at P_0 is: $T_0 = 273\ K$
   The tank is considered as closed and rigid, therefore $V_0=V_{ma\cdot}$
   With the Eq. 1-3 : $P_0 \cdot V_0 / T_0 = P_{ma\cdot} \cdot V_{ma\cdot} / T_{ma\cdot}$
   => $T_{ma\cdot} = P_{ma\cdot} \cdot T_0 / P_0 = 12 \times 10^5 \times 273 / 10 \times 10^5 = 327.6\ K$ or $T_{ma\cdot} = 54.6\ °C$
   The maximum pressure could be easily attained through temperatures that could realistically occur in a confined enclosure.

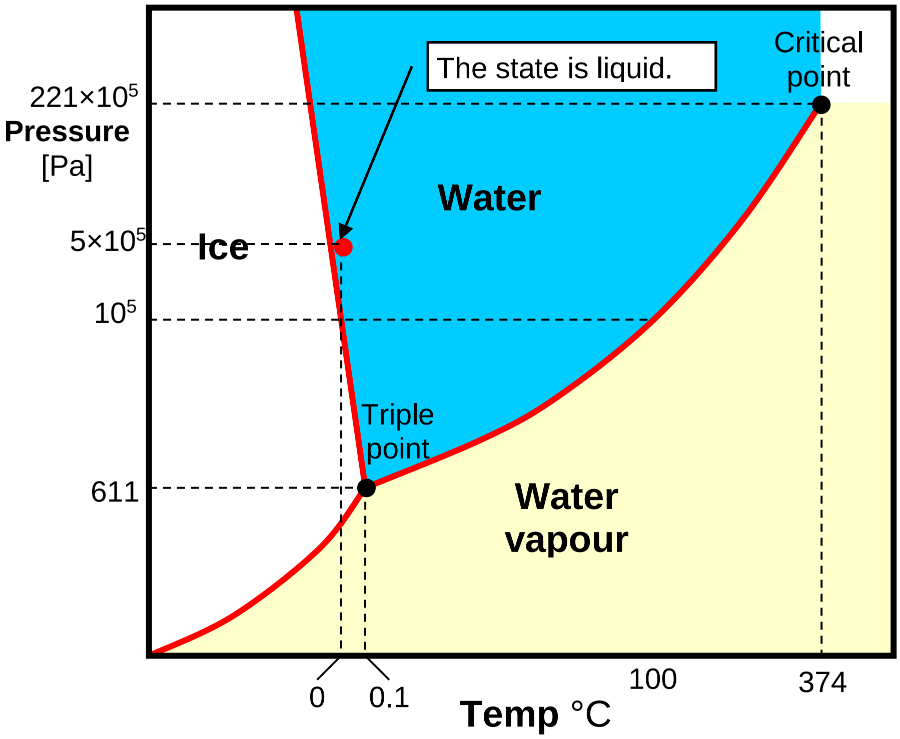
5. **What is the water density under an increase of $30 \cdot 10^5$ Pa at 5°C?**
   The mass of 1 m^3 of water is 1000 kg
   The bulk modulus for water is: $K_{Water} = 2.2 \times 10^9\ Pa$
   The volume of 1 m^3 of water under 30×10^5 Pa will decrease according to Eq.1-4
   => $\Delta V = \Delta P \cdot V_0 / K_{Water} = 30 \times 10^5 \times 1 / 2.2 \cdot 10^9 = 0.0014\ m^3$
   The volume of 1000 kg of water at $30 \cdot 10^5$ Pa is therefore 1-0.0014 = 0.9986 m^3
   The density according to eq 1.1 $\rho = m / V = 1000 / 0.9986 = 1\ 001.4\ kg/m^3$
   Note that the increase of density due to a huge pressure (30×10^5 Pa) has a similar effect as an increase of 15° of temperature.

Answers to intermediary exercises

6. **What is the reduction of volume of a 1m^3 steel cube under an increase of $20 \cdot 10^5$ Pa?**
   The variation of pressure is: $\Delta P = 20 \times 10^5\ Pa$
   The bulk modulus for steel is: $K_{steel} = 160 \times 10^9\ Pa$
   The initial volume is: $V_0 = 1\ m^3$

With the Eq. 1-4 we find: $\Delta V = \Delta P \cdot V_0 / K_{Steel} = 20 \times 10^5 \times 1 / (160 \times 10^9) = 12.5 \times 10^{-6}\ m^3 = 12.5\ cm^3$
The maximum pressure could be reached with a temperature that could be easily found in certain condition like in a closed building without aeration.

7. What is the phase of the water at $5 \cdot 10^5$ Pa and 0°C?



8. What will be the length and diameter of a PE pipe (original length 20m, original diameter 200mm), after thermal expansion due to temperature changing from 5°C to 25°C?

For the length, we have the following formula: $\Delta L = L \cdot \Delta T \cdot \alpha_T$
the length is $L = 20$ m
the temperature variation is $\Delta T = 25 - 5 = 20°$
the thermal expansion coefficient for PE is: $\alpha_T = 0.2$ mm/m°K
$\Delta L = 20 \times 20 \times 0.2 = 80$ mm $= 0.08$ m
the final length is $L_f = L_i + \Delta L = 20 + 0.08 = 20.08$ m

For the diameter, we have $\Delta D = D \cdot \Delta T \cdot \alpha_T$
the diameter is $L = 0.1$ m
the temperature variation is $\Delta T = 25 - 5 = 20°$
the thermal expansion coefficient for PE is: $\alpha_T = 0.2$ mm/m°K
$\Delta D = 0.1 \times 20 \times 0.2 = 0.4$ mm
Therefore, the final diameter is $D_f = D_i + \Delta D = 100 + 0.4$ mm $= 100.4$ mm
In fact, the thermal expansion of the diameter, is negligible, since it is almost as much as the error margin for PE pipe.

9. A plate 50 x 50 cm is supported by a water layer 1 mm thick. What force must be applied to this plate so that it reaches a speed of 2m/s at 5°C and at 40°C?

The viscosity of water at 5°C is according to table 1: $\mu_{5°C} = 1.519 \times 10^{-3}$ Pa·s
The viscosity of water at 20°C is according to table 1: $\mu_{20°C} = 0.661 \times 0.992 \cdot 10^{-3} = 0.656 \times 10^{-3}$ Pa·s
The surface of the plate is: $A = 0.5 \times 0.5 = 0.25\ m^2$
The velocity is: $v = 2$ m/s
The distance between the plates is: $y = 0.001$ m

With the Eq. 1-6 we find the force to be applied:

at 5°C : $F = 1.519 \times 10^{-3} \times 0.25 \times 2 / 0.001 = 0.760$ N

at 40°C: $F = 0.656 \times 10^{-3} \times 0.25 \times 2 / 0.001 = 0.328$ N

The viscosity of the water is quite small, as with a force of "76 grams" we can move the plate and at 40°C half of this force is necessary.

Answers to the advanced exercises

10. What is the % increase of diameter of PE pipe of outside diameter 200 mm, and thicknesses of 9.1 mm, 14.7 mm, 22.4 mm, at $10 \cdot 10^5$ Pa?

The variation of pressure is: $\Delta P = 10 \times 10^5$ Pa

The bulk modulus for PE is: $K_{PE} = 1.2 \times 10^9$ Pa

The diameter is: D = 0.2 m

The thicknesses are: $e_1 = 0.0091$ m, $e_2 = 0.0147$ m, $e_3 = 0.0224$ m

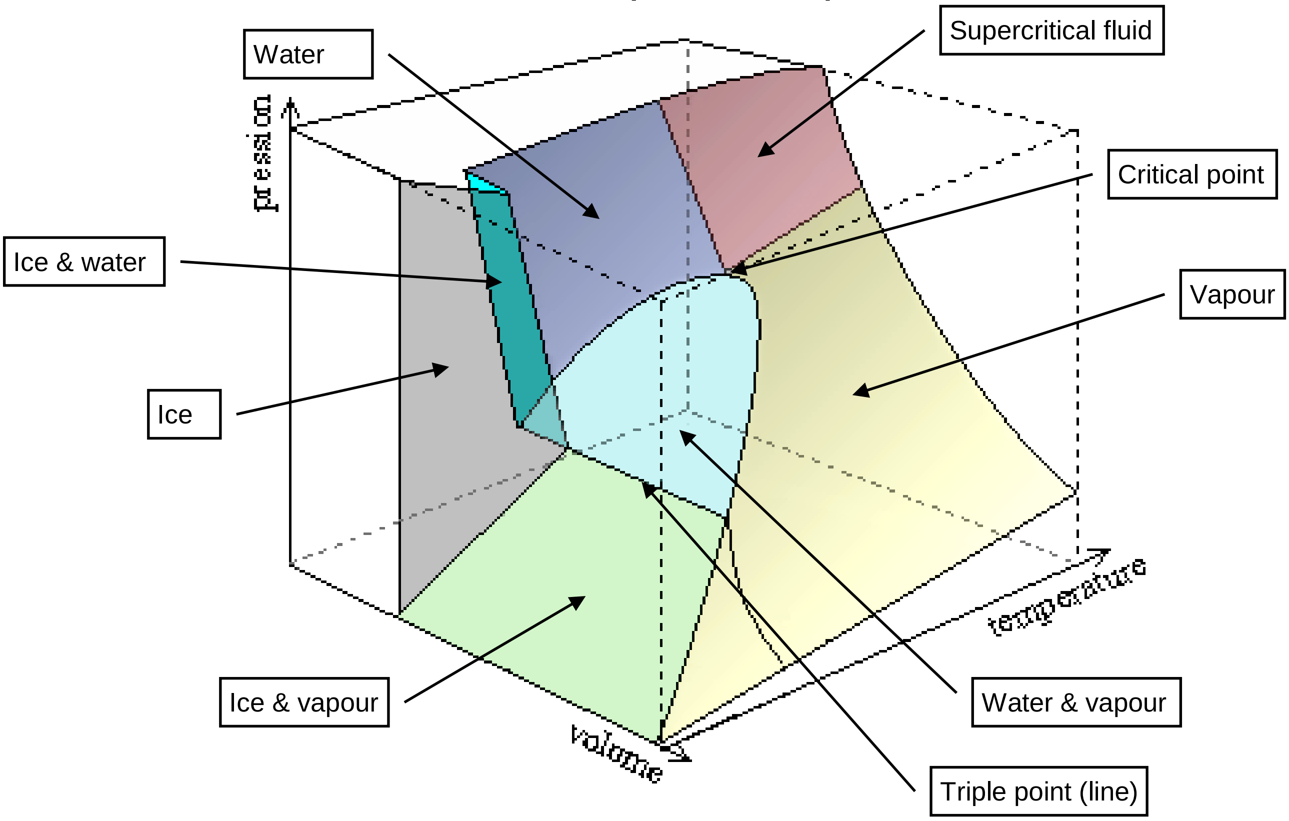
With the Eq. 1-5 we find:

- With e_1 : $\Delta D / D = 10 \times 10^5 / (1.2 \times 10^9) \times 0.2 / (2 \times 0.0091) = 0.9$ %
- With e_2 : $\Delta D / D = 10 \times 10^5 / (1.2 \times 10^9) \times 0.2 / (2 \times 0.0147) = 0.6$ %
- With e_3 : $\Delta D / D = 10 \times 10^5 / (1.2 \times 10^9) \times 0.2 / (2 \times 0.0224) = 0.4$ %

The increase of the pipe diameter even with the thinner one is smaller than the construction tolerance.

11. the following three dimensional chart (P,T,V) representation of the phase element find the following :

- **Critical point & triple point (line)**
- **Plain: water; ice; vapour; supercritical fluid**
- **Mixed: water & ice; water & vapour; ice & vapour**



Answers to basic exercises

1. **What is roughly the pressure an 80kg man exerts on the ground?**

   From **Eq2-1**, the pressure is given by $P = F / A$
   In this case, F is the gravity force: $F = m \cdot g = 80 \times 9.81 = 784.8$ N
   The surface A is the sole of the 2 feet. So approx $A = 2 \times 0.25 \times 0.15 = 0.075$ m^2
   So P = 10 464 Pa.
   Only 10 % additional to the Atmospheric Pressure (101 325 Pa), it might change a lot with the shape of the soles.

2. **What is the water pressure at the bottom of a pool 15 m deep at 5°C and at 25°C?**

   From **Eq2-6**, $P_{bottom} = \rho \cdot g \cdot h$
   At 5°C, $\rho = 1000$ kg/m^3, P bottom = 147 150 Pa
   At 25°C, $\rho = 997.1$ kg/m^3, P bottom = 146 723 Pa
   At both temperatures, it remains close to 15 mWC

Answers to intermediary exercises

3. **What is the average atmospheric pressure at your place and at 4'000 masl?**

   At sea level (0 masl), the atmospheric pressure is 1 013 hPa.
   In Pyong Yang, elevation is 84masl.
   From **Eq2-3**, Pressure (84 masl) = 1 003 hPa, so 1% variation from the pressure at sea level.
   At 4 000 masl, Pressure (4'000 masl) = 616 hPa, so 39% variation from the pressure at sea level.

4. **At 4'000 masl, what is the elevation of a column of mercury and a column of water?**

   From **Eq2-2**, the height of the mercury column is given by: $h = Pa / \rho \cdot g$
   As seen in question 2, the Pa at 4 000 m is : Pa = 61 645. g is assumed as 9.81 m/s^2
   With mercury, density ρ is 13 500 kg/m^3. At 4 000 masl, h = 0.47 m
   For water, if the effect of the vapour pressure is neglected (at low temperature) the same equation can be used.
   With water, density is 1 000 kg/m^3. At 4 000 masl, h = 6.28 m. To be compared with 10 m at sea level !

5. **What is the maximum suction height for water at 30°C at 1'500 masl?**

   From **Eq2- 3,** Pa (1500 masl) = 845.59 hPa (83% of Pa at sea level)
   From **the graph,** $Pv = 0.00000942 \cdot t^4 - 0.000291 \cdot t^3 + 0.0331 \cdot t^2 + 0.242 \cdot t + 6.11$
   Pv (30°) = 43 hPa
   From **Eq2-4,** the maximum pumping height is given by: $h = (Pa\text{-}Pv) / \rho(30°) \cdot g$
   so $h = (84\,559 - 4\,300) / (995.7 \times 9.81) = 8.22$ m

6. **What is the maximum suction height for water at 15°C at 3'000 masl?**

   Following the same path than question 5, we have Pa (3'000 masl) = 701.12 hPa

   Pv (15°) = 17 hPa. ρ (15°) = 999.1. Thus h = 6.98 m

7. **What is the buoyant force applied to a body of 2 m³?**

From **Eq2.7**, $F = \rho \cdot g \cdot V$
The nature of the body is not of importance but the nature of fluid is important
If it is water, ρ is 1 000 Kg/m^3, F = 1000 × 9.81 × 2 F= 19 620 N
If it is mercury, ρ is 13 500 Kg/m^3, F = 13500 × 9.81 × 2 F= 264 870 N

Answers to the advanced exercises

8. **A water pipe has its top at 500 mASL and its bottom at sea level. What is the variation of atmospheric pressure between the top and the bottom? What is the water pressure at the bottom? What is the ratio between these two pressures?**

From **Eq2-3,** Patm(0 mASL) = 101 325, Patm(500 mASL) = 95 462
=> ΔPatm (0 to 500 mASL) = 5 863 Pa
From **Eq2-5,** Pwater = $\rho \cdot g \cdot h$ = 1 000 × 9.81 × 500 = 4 905 000 Pa
The **ratio** is Pwater / ΔPatm = 836
The water pressure is much more important than the variation of air pressure, therefore difference of atmospheric pressure due to the variation of altitude can be neglected in water systems.

9. **What is the force applied by the liquid to the side of a water tank that is 2 m high and 10 m wide?**

The pressure is nil at the top and increases linearly till the maximum at the bottom. The average can therefore be taken at the centre.
From **Eq2-6,** $P = \rho \cdot g \cdot h$
The centre of the side of the tank is at 1m down. So at 1m depth, Pw = 9.81×10^3 Pa
From **Eq2-1,** P =F/A. Thus F = P · A.
The surface of the side of the tank is A = 10 × 2 m^2.
At the centre of the side of the tank, F = 196.20 kN
It weighs as a truck of 19.6 tons.

10. **What is the buoyant force exerted on an 80 kg man by the atmosphere?**

From **Eq2-7**, $F = \rho \cdot g \cdot V$
In Chapter 1, question 3, we have calculated the density of air at 1×10^5 Pa and 30°C.
=> ρ = 1.15 kg/m3
The density of a human body is close to the water density (as we are almost floating in water), thus **Eq1-1** => $V = m / \rho = 80 / 10^3 = 0.08\ m^3$
With g = 9.81 m/s^2, the buoyant force from atmosphere is F = 1.15 × 9.81 × 0.08 = 0.90 N
The gravity force is F = m · g = 784.8 N …That is why we do not float or fly !

11. **An air balloon of 1m3 at atmospheric pressure is brought at 10m below water surface, what will be the buoyant force? And at 20m below water surface?**

The volume of the balloon can be calculated with the ideal gas law (**Eq. 1-3**) the temperature can be considered as constant thus $P \cdot V$ = constant. At 10m below water surface the pressure is twice the atmospheric pressure thus the volume will be half:
=> $V_{10m} = V_{surface} \cdot P_{surface} / P_{10m} = 1 \times 1 / 2 = 0.5\ m^3$
From **Eq2-7**, $F = \rho \cdot g \cdot V = 10^3 \times 9.81 \times 0.5 = 4\,905$ N
At 20m below water surface, the water pressure will be about 2 atm, and total pressure 3 atm thus:
=> $V_{20m} = V_{surface} \cdot P_{surface} / P_{20m} = 1 \times 1 / 3 = 0.333\ m^3$
From **Eq2-7**, $F = \rho \cdot g \cdot V = 10^3 \times 9.81 \times 0.333 = 3\,270$ N

Answers to basic exercises

1. **What is the flow in a pipe of 150mm of diameter with a 1m/s speed?**

   The section area is: $A = \pi \cdot D^2 / 4 = 3.1415 \times (0.15)^2 / 4 = 0.01767$ m2
   The mean velocity is: $v = 1$ m/s
   With the **Eq. 3-2** the flow is: $Q = A \cdot v = 0.01767 \times 1 = 0.01767$ m3/s = 17.7 l/s

2. **What is the speed in the same pipe after a reduction in diameter to 100mm and to 75mm?**

   From **Eq. 3-2**, $Q = A \cdot v = (\pi D^2 /4) \cdot v$ and Q is constant $Q_1 = Q_2 = A_2 \cdot v_2$
   After reduction to a pipe of 100 mm of diameter, $v_2 = Q_1 / A_2$. So $v_2 = 2.25$ m/s
   After reduction to a pipe of 75 mm of diameter, $v_3 = Q_1 / A_3$. So $v_3 = 4$ m/s.
   With a reduction of 50% in diameter, the speed is increased by 4.

3. **What is the flow in the same pipe after a Tee with a 25mm pipe with a 1 m/s speed?**

   From **Eq. 3-3,** $Q_1 = Q_{tee} + Q_2 \Rightarrow Q_2 = Q_1 - Q_{tee}$
   $Q_{tee} = (\pi D^2 /4) \cdot v$ so $Q_{tee} = 0.000491$ m^3/s = 0.49 l/s
   So the flow after the tee will be $Q_2 = 17.7 – 0.49 = 17.2$ l/s

4. **What is the speed of water going out of the base of a tank with 2.5m, 5m and 10m height?**

   If we neglect the losses, applying the Law of Conservation of Energy, the speed of the water is given by $v = \sqrt{2 \cdot g \cdot h}$
   For $h = 2.5$ m : $v = 7$ m/s
   For $h = 5$ m : $v = 9.9$ m/s
   For $h = 10$ m : $v = 14$ m/s.
   When multiplying the height by 4, the speed is multiplied by 2 only.

5. **What are the speed and the flow in a pipe of 150mm of diameter showing a difference of 20cm height in a Venturi section of 100mm diameter?**

   In **Eq. 3-6,** $A_1/A_2 = (D_1/D_2)^2$ with $D_1 = 0.15$ m, $D_2 = 0.10$ m & $h = 0.20$m
   $\Rightarrow v_1 = \sqrt{(2 \times 9.81 \times 0.2 / ((0.15 / 0.1)^4-1))} = 0.983$ m/s
   From **Eq. 3-2**, $Q_1 = (\pi D_1^2 /4) \cdot v_1 = 3.1415 \times 0.15^2 / 4 \times 0.983 = 17.36$ l/s

Answers to intermediary exercises

6. **What are the Reynolds numbers for the flows in the exercises 1,2 & 5?**

   From Eq 3.7 Re = D · v /*v*

D (m)	v (m/s)	ν (m²/s)	Re
0.150	1.00	1.E-06	150 000
0.100	2.25	1.E-06	225 000
0.075	4.00	1.E-06	300 000
0.150	0.98	1.E-06	147 450

It can be noted that we are always in turbulent conditions.

7. For a pipe of 150mm of diameter, with 2 bar pressure a velocity of 1 m/s, what should be the diameter reduction to cause cavitation at 20°C (neglect head losses)?

With Eq 3-4, as the elevation is the same, $H_1 = H_2$, we can neglect the head losses and dived all terms by g:

$$\frac{P_1}{\rho}+\frac{v_1^2}{2}=\frac{P_2}{\rho}+\frac{v_2^2}{2}\Rightarrow\frac{P_1-P_2}{\rho}=\frac{v_2^2-v_1^2}{2}$$

Furthermore, with Eq 3-2, we found v_2 according to v_1 :

$$Q=v_iA_i=cte\Rightarrow v_1A_1=v_2A_2\Rightarrow v_2=v_1\frac{A_1}{A_2}$$

To have cavitation, the total pressure should be at P_v, i.e. the water pressure should the vapour pressure (2 400 Pa at 20°C).

$$\frac{P_1-P_2}{\rho}=\frac{v_2^2-v_1^2}{2}\Rightarrow\frac{P_1-P_2}{\rho}=\frac{v_1^2\left(\left(\frac{A_1}{A_2}\right)^2-1\right)}{2}\Rightarrow\sqrt{\frac{2\cdot(P_1-P_2)}{\rho\cdot v_1^2}+1}=\frac{A_1}{A_2}$$

$$\frac{A_1}{A_2}=\frac{D_1^2}{D_2^2}\Rightarrow D_2=D1\sqrt{\frac{A_2}{A_1}}$$

P_1 = 200 000Pa, P_2 = 2 400 Pa

We find A_1/A_2 = 19.9 and D2 = 33.6 [mm]

Answers to advanced exercises

8. With equations Eq. 2-5, Eq. 3-2 & Eq. 3-4, demonstrate Eq. 3-6.

As for the previous exercice, with Eq 3-4, as the elevation is the same, $H_1 = H_2$, we can neglect the head losses and dived all terms by g, we have:

$$\frac{P_1}{\rho}+\frac{v_1^2}{2}=\frac{P_2}{\rho}+\frac{v_2^2}{2}\Rightarrow\frac{P_1-P_2}{\rho}=\frac{v_2^2-v_1^2}{2}$$

With Eq 3-2, we found v_2 according to v_1 : $Q=v_iA_i=cte\Rightarrow v_1A_1=v_2A_2\Rightarrow v_2=v_1\frac{A_1}{A_2}$

With Eq 2-5, we found the difference of pressure:

$$P_{tot}=P_{atm}+\rho\cdot g\cdot h\Rightarrow P_1-P_2=\rho\cdot g\cdot(h_1-h_2)=\rho\cdot g\cdot\Delta h$$

Inserting these elements in the first equation:

$$\frac{\rho\cdot g\cdot\Delta h}{\rho}=\frac{\left(v_1\frac{A_1}{A_2}\right)^2-v_1^2}{2}\Rightarrow 2\cdot g\cdot\Delta h=v_1^2\left(\left(\frac{A_1}{A_2}\right)^2-1\right)$$

Thus we find Eq 3-6 $v_1=\sqrt{2\cdot g\cdot\Delta h\Big/\left(\left(\frac{A_1}{A_2}\right)^2-1\right)}$

9. For a pipe of 150mm of diameter, with 1 bar pressure a velocity of 1 m/s, what should be the diameter reduction to cause air suction?

We suppose that elevation is the same, thus $H_1=H_2$ and losses are neglected.
To have air suction, the total pressure should be at Patm, i.e. the water pressure should be nil. Similar formulas as for the previous exercise give the following:

$$\frac{P_1-P_2}{\rho}=\frac{v_2^2-v_1^2}{2}\Rightarrow\frac{P_1-P_2}{\rho}=\frac{v_1^2\left(\left(\frac{A_1}{A_2}\right)^2-1\right)}{2}\Rightarrow\sqrt{\frac{2\cdot(P_1-P_2)}{\rho\cdot v_1^2}+1}=\frac{A_1}{A_2}$$

$$\frac{A_1}{A_2}=\frac{D_1^2}{D_2^2}\Rightarrow D_2=D_1\sqrt{\frac{A_2}{A_1}}$$

Thus with P_1= 100 000 Pa, v_1 =1 m/s, we find A_1/A_2 = 14 and D_2 = 40 [mm]
In reality the head losses can not be neglected and the actual value will be bigger, this will be seen in the next chapter.

10. A tank of 2m high and 1m diameter has a 75mm valve at the bottom. When the tank is full and the valve is quickly opened, how long does it take to empty the tank (the losses and contraction factor are neglected)?

There is 2 ways to resolve this exercise

A) Through a differential equation:

The flow going out of the tank is: $Q=A_{valve}\sqrt{2gh}$

The variation of height of the water level in the tank is: $\partial h=-\frac{Q}{A_{Tank}}\partial t$

Combining the two equations and rearranging: $\partial h=-\frac{A_{valve}\sqrt{2gh}}{A_{Tank}}\partial t\Rightarrow\frac{-A_{Tank}}{A_{valve}\sqrt{2gh}}\partial h=\partial t$

This equation can be integrated: $\int_{t1}^{t2}\partial t=\int_{h1}^{h2}\frac{-A_{Tank}}{A_{valve}\sqrt{2gh}}\partial h=\frac{-A_{Tank}}{A_{valve}\sqrt{2g}}\int_{h1}^{h2}h^{-1/2}\partial h$

The final results is then $t=t_2-t_1=\frac{-A_{Tank}\cdot 2\cdot\left(\sqrt{h_2}-h_1^{1/2}\right)}{A_{valve}\sqrt{2g}}=\frac{A_{Tank}\cdot\sqrt{2}\cdot\left(h_1^{1/2}-h_2^{1/2}\right)}{A_{valve}\sqrt{g}}$

Thus with h_2=0, h_1=2, A_{tank} = 0.785, A_{valve}=0.0044 we find t = 114 s

B) Easier way but only working if the diameter of the tank is constant:

It will take the time to empty the tank at the average velocity

$$V_{avg}=\frac{V_1-V_2}{2}\Longrightarrow V2=0\Rightarrow V_{avg}=\frac{\sqrt{2gh}}{2}$$

Answers to basic exercises

1. What are the ID, the SDR and the Series of a pipe with an outside diameter of 110 mm and a thickness of 6.6 mm?

Using OD = 110 mm and e = 6.6mm
From **Eq. 4-2**, ID = OD – 2e   ID = 110 - 2 × 6.6,   ID = 96.8 mm
From **Eq. 4-3**, SDR = OD / e   SDR = 110 / 6.6   SDR = 16.67 rounded to 17
From **Eq. 4.4**, S = (SDR-1)/2   S = ((110/6.6) -1)/2   S = 7.83 rounded to 8

2. What is the nominal pressure of this pipe if it is made with PVC and used at a temperature of 30°c? If it is made with PE80, PE100 at 20°C?

From Eq 4.5, PN = 10 MRS / (S · C)
For a diameter 110 in PVC, MRS = 25 MPa and C=2
For temperature lower that 25 °C, PN = 10 (25/ (8×2) PN = 16 bar
But 30°C, the derating temperature factor (ft) is = 0.88
So finally, PN is only 14 bar.

For a diameter 110 in PE 80 at 20°C, MRS = 8 and C= 1.25, (no derating factor).
From **Eq 4.5**   PN = 10 × 8 / (8 × 1.25)   PN = 8 bar

For a diameter 110 in PE 100 at 20°c, MRS = 10 and C= 1.25, (no derating factor).
From **Eq 4.5**   PN = 10 × 10 / (8 × 1.25)   PN = 10 bar

3. What is the nominal pressure of these PE pipes if it is used for gas (the service ratio is 2 for gas)?

For a diameter 110 in PE 80 at 20°c, MRS = 8 and C= 2, (no derating factor).
From Eq 4.5   PN = 10 × 8 / (8 × 2) PN = 5 bar

For a diameter 110 in PE 100 at 20°c, MRS = 10 and C= 2, (no derating factor).
From Eq 4.5   PN = 10 × 10 / (8 × 2)PN = 6 bar

4. What are the friction losses in a DN150 PVC pipe of 2.2 km, with a velocity of 1 m/s? Same question for a cast iron pipe (k=0.12mm) (can be estimated with figures: either log scale of charts in annexes)?

PVC pipe:

With the charts (taking the chart for k=0.005), for a pipe of diameter 150mm and a velocity of 1 m/s the result is 0.56 m/100m. Given that we have a pipe 2.2 km long, $H_L = 0.56 \times 22 = 12.3$ m

Steel pipe:

With the charts (taking the chart for k=0.12), for a pipe of diameter 150mm and a velocity of 1 m/s the result is 0.7 m/100m. Given that we have a pipe 2.2 km long, $H_L = 0.7 \times 22 = 15.4$ m

Answers to intermediary exercises

5. What is the hydraulic diameter of a square pipe (b=h)? Of a flatten (elliptic) pipe with b=2h?

For the square pipe: From **Eq 4.7**, $A = h^2$ and $P = 4h$, so in **Eq 4.6**, $D_h = 4\ A/P$, $D_h = h$

For the flatten pipe: From **Eq 4.8** $A = \pi\ h^2 / 2$ and $P = 3\ \pi\ h / 2$, so $D_h = 4\ A/P$, $D_h = 4h / 3$

6. What is the minimum velocity and flow that should flow to avoid air pocket (Re = 10'000) in a pipe of DN25, DN200, DN500 ?

From **Eq = 4.9**, Re = $D \cdot v / \nu$ so, $v = Re \cdot \nu / D$
When Re = 10'000, the minimum velocity is reached
When D = 0.025 m, v = 10'000 × 0.000001 /0.025 v = 0.4 m/s
The flow will then be : $Q = \pi D^2 \cdot v / 4$ $Q = 3.14 \times (0.025)^2 \times 0.4 / 4$ Q= 0.196 l/s
When D = 0.2 m, v = 10'000 0.000001 /0.2 v = 0.05 m/s
The corresponding flow will be : 1.57 l/s
When D = 0.5 m, v = 10'000x 0.000001 /0.5 v = 0.02 m/s
The corresponding flow will be : 3.92 l/s

7. **What is the punctual friction losses coefficient for a pipe connected between to tanks, with four round elbows (d=D), a gate valve, a non-return valve and a filter?**

- 4 round elbows (d=D), $K_p = 4 \times 0.35$
- 1 gate valve, $K_p = 0.35$
- 1 non-return valve, $K_p = 2.5$
- 1 filter, $K_p = 2.8$
- 1 inlet, $K_p = 0.5$
- 1 discharge, $K_p = 1.5$

The punctual friction losses coefficient is the sum, $K_{ptotal} = 9.05$

8. **What is the average punctual friction losses coefficient for the accessories of a DN200 pump?**

A classical setup includes the following items:

- foot valve with strainer, $K_p = 15$
- 3 round elbows (d=D), $K_p = 3 \times 0.35$
- 1 reduction (d/D=0.8 45°), $K_p = 0.15$
- 1 extension (d/D=0.8 45°), $K_p = 0.1$
- 1 gate valve, $K_p = 0.35$
- 1 non-return valve, $K_p = 2.5$

The punctual friction losses coefficient is the sum, $K_{ptotal} \approx 20$, three quarter are due to the foot valve, this is usually the most critical part.

9. **What are the friction losses in a DN150 PVC pipe of 2.2 km, with a velocity of 1 m/s? Same question for a steel pipe (k=1mm) (to be calculated with equations, not estimated with charts)?**

The Reynolds number is $Re = D \cdot v / \nu$, Re = 150'000
For a PVC pipe, the flow will be turbulent smooth
From **Eq 4.18**, $\lambda = 0.309 / (\log (Re/7))^2 = 0.0165$.
From **Eq 4.15** and $k_L = \lambda\ L/D = 0.0165 \times 2\,200/0.15 = 241.6$
From **Eq 4.10** and $H_L = k_L v^2/2g = 12.3$ m
If we compare this result with the one obtained in exercise 4, we see that the results match approximately. However, the method used in exercise 4 are usually less precise.

For a DN 150 steel pipe, the flow will be turbulent partially rough.
The ratio k/D = 1/150 = 0.007
From **Eq 4.17**, $\lambda = 0.0055 + 0.15/(D/k)^{(1/3)} = 0.033$ with **Eq 4.19** $\lambda = 0.034$ thus taking rough instead of partially rough underestimate the losses.
From **Eq 4.15** and $k_L = \lambda\ L/D = 0.034 \times 2\,200/0.15 = 498$
From **Eq 4.10** and $H_L = k_L v^2/2g = 25.4$ m

10. **What should be the diameter of an orifice to create losses of 20 meters in a DN100 pipe with a velocity of 1 m/s?**

For an orifice we can use **Eq 4-14,** $K_{p\text{-}o} = H_{P\ Orifice} \cdot 2g / v_1^2$
with $H_{P\ Orifice}$ = 20 m and v_1 = 1m/s, $K_{p\text{-}o} = 391 \approx 400$.

On the chart, d/D is :

- if it is a sharp orifice : d/D = 0.28 which make d = 2.8 cm
- if it is a beweled orifice : d/D = 0.27 which makes d= 2.7 cm
- if it is a rounded orifice d/D = 0.24 which makes d= 2.4 cm

Practically a hole of 2.4 cm should be done in plate placed between two flanges. This orifice can then be filed down to be increased if necessary.

Answers to advanced exercises

11. What is the flow in a DN150 galvanised pipe of 800 m connecting two tanks with a gate valve, five elbows and a filter, if the difference of height between the tanks is 2m; 5m; 10m?

From **Eq 4-11** we have: $H_A - H_B = \Delta H = (Q^2 (K_p + K_l)) / (12.1 \times D^4)$
with $D = 0.15 \Rightarrow Q^2 = \Delta H \times 0.00613 / (K_p + K_l)$

To calculate $K_{ponctual}$ it is necessary to sum all punctual losses, the system is composed of:

System	K_p
1 inlet	0.5
1 gate valve	0.35
5 elbows (round)	1.75
1 filter	2.8
1 discharge	1.5
Total	$K_{ponctual} = 6.9$

To estimate K_{linear} we calculate the ratio k/D = 0.001 (D = 150mm and k = 0.15 m). We can see in the Moody chart that with this ratio we have a λ between 0.04 and 0.02 according to the Reynolds. Let assume a value of 0.02 (Re > 800'000, turbulent rough) to start. Then from **Eq. 4-15,** $K_{linear} = \lambda L/D = 0.02 \times 800/0.15 = 106.7$

Then we can calculate the flow: $Q^2 = 2 \times 0.00613 / 113.6 \Rightarrow Q = 0.0104\ m^3/s$ or 10.4 l/s
The velocity and Reynolds can now be calculated:
$v = 4 \cdot Q / (\pi \times D^2) = 0.59$ m/s and $Re = D \cdot v / \nu = 88'000$

By checking the Moody chart, we can see that the λ has roughly a value of 0.023, giving a value of $K_{linear} = 122.7$, what is not negligible. The calculation should be done again with this new value giving:
Q= 0.0097 m^3/s or 9.7 l/s, v=0.55 m/s and Re = 83'000
This time the Re is close enough to our estimation, one iteration was enough to find a precise value

For ΔH = 5m, with a λ= 0.02, we find with the first calculation
Q= 0.0164 m^3/s or 16.4 l/s, v=0.93 m/s and Re = 140'000
For a Re of 140'000 lets take a λ=0.022 and recalculate:
Q= 0.0157 m^3/s or 15.7 l/s, v=0.89 m/s and Re = 133'000 what is precise enough

For ΔH = 10m, with a λ= 0.02, we find with the first calculation
Q= 0.0232 m^3/s or 23.2 l/s, v=1.31 m/s and Re = 197'000
For a Re of 197'000 lets take a λ=0.021 and recalculate:

Q= 0.0227 m^3/s or 22.7 l/s, v=1.28 m/s and Re = 192'000 what is precise enough

12. How much shall a pipe be crushed to reduce the flow by half?

With equation 4-11 we know that $H_A - H_B = \sum H_{LP} \cong \frac{Q^2}{12.1} \cdot \sum \frac{k_{LP}}{D^4}$ (1)

Given that H_A-H_B is constant, we have $\frac{(Q_1/2)^2}{12.1} \cdot \sum \frac{k_{LP}}{{D_1}^4} = \frac{(Q_2)^2}{12.1} \cdot \sum \frac{k_{LP}}{{D_2}^4}$ (2)

We assume that λ does not vary significantly with the change occurring after the crushing. Furthermore, we have $k_{LP} = \lambda \cdot L/D$ (3). Putting (3) into (2) and multiplying both sides by 12.1 we get

$\frac{{Q_1}^2}{4} \cdot \frac{\lambda \cdot L}{D_1^5} = {Q_2}^2 \cdot \frac{\lambda \cdot L}{D_2^5}$ (4) Which gives us $\frac{1}{D_1^5} = \frac{4}{D_2^5}$ (5) so at the end $\frac{D_1}{D_2} = \left(\frac{1}{4}\right)^{1/5} = 0.758$ (6). It is important to notice that while D_1 is the nominal diameter, D_2 is the hydraulic diameter D_h (since it is an ellipse). Therefore, for the rest of the exercise D_N and D_h will be used for D_1 and D_2 respectively.

Now we want to find h as a function of D.
For an ellipse (the conduct is crushed in an ellipsoidal shape), we have (eq 4-7)

$A = \frac{\pi}{4} \cdot b \cdot h$ (7) and $P = \frac{\pi}{2} \cdot (b + h) = \pi \cdot D_N$ (8) because the perimeter of the conduct does not change if the pipe is crushed. From this, it follows that $b + h = 2 \cdot D_N$ or $b = 2 \cdot D_N - h$ (9) substituting (9) into (7) we get

$A = \frac{\pi}{2} \cdot D_N \cdot h - \frac{\pi}{4} \cdot h^2$ (10)

Using equation 4-6 we know that $A = \frac{D_h \cdot P}{4}$ (11) where D_h is in this case D_2 or D_h=0.758D_N (12) Substituting (8) and (12) into (11) we get $A = \frac{0.758 \cdot D_N \cdot \pi \cdot D_N}{4}$ (13)

Given that (13)=(10) we get $\frac{\pi}{2} \cdot D_N \cdot h - \frac{\pi}{4} \cdot h = \frac{0.758 \cdot D_N \cdot \pi \cdot D_N}{4}$ (14)

Dividing (14) by π and multiplying by 2 we get

$0.758 \cdot D_N^2 = 2 \cdot D_N \cdot h - h^2$ which gives us a second grade equation $h^2 - 2 \cdot D_N \cdot h + 0.758 \cdot D_N^2 = 0$ (15) solving this equation we have

$h = \frac{2 \cdot D_N \pm \sqrt{4 \cdot D_N^2 - 4 \cdot 0.758 \cdot D_N^2}}{2}$ (16) or $h = D_N \cdot (1 \pm 0.492)$ (17). So the two solutions for h are:

$h_1 = 1.492 \cdot D_N$ and $h_2 = 0.508 \cdot D_N$ where D_N is nominal diameter of the uncrushed pipe. These two solutions correspond to the pipe being crushed horizontally or vertically.

13. What is the flow when the pipe is crushed to half of its diameter?

It is the same approach as for ex 11, but the other way around.

If the pipe is crushed to half its diameter, the height of the ellipse (h) will be $h = \frac{1}{2} \cdot D_N$ (1) As before, we have $A = \frac{\pi}{4} \cdot b \cdot h$ (2) and $P = \frac{\pi}{2} \cdot (b + h) = \pi \cdot D_N$ (3) which gives us $A = \frac{\pi}{2} \cdot D_N \cdot h - \frac{\pi}{4} \cdot h^2$ (4). Substituting (1) into (4) we get $A = \frac{\pi}{4} \cdot D_N^2 - \frac{\pi}{16} \cdot D_N^2 = \frac{3 \cdot \pi}{16} \cdot D_N^2$ (5) Knowing from ex 11 that $A = \frac{D_h \cdot P}{4} = \frac{D_h \cdot \pi \cdot D_N}{4}$ (6) We can set that (5) equal (6). So $\frac{D_h \cdot \pi \cdot D_N}{4} = \frac{3 \cdot \pi}{16} \cdot D_N^2$ (7) It follows that $D_h = \frac{3}{4} \cdot D_N$ (8). As in ex 11, we have $Q_1^2 \cdot \frac{\lambda \cdot L}{D_1^5} = Q_2^2 \cdot \frac{\lambda \cdot L}{D_2^5}$ (9) Where D_2 is D_h and D_1 is D_N. Substituting (8) into (9) we have $Q_1^2 \cdot \frac{\lambda \cdot L}{D_N^5} = Q_2^2 \cdot \frac{\lambda \cdot L}{\left(\frac{3}{4}\right)^5 \cdot D_N^5}$ (10) and it follows that $\frac{Q_2}{Q_1} = \left(\frac{3}{4}\right)^{5/2}$ (11) or

$Q_2 = 0.487 \cdot Q_1$ (12)

If we compare this result with what we obtained in ex 11 we see that the result we get here fits to what we calculated previously. Indeed, in ex 11, in order to reduce the flow by half, we had to reduce pretty much the diameter by half. Here, if we reduce the diameter by half, we reduce the flow by almost half of it.

Answers to basic exercises

1. **What is the hydraulic radius of a square channel with the same width as depth (h=b)?**

   From **Eq 5-3,** $A = b \cdot h = b^2$ and $P = b+2h = 3b$
   From **Eq 5-2,** $R_h = A/P = b^2 /3b$ $R_h = b/3$

2. **What is the hydraulic radius of a half full pipe (h=D/2), of a full pipe (h=D)?**

   For a half full pipe (h=D/2), $\alpha = 180° = \pi$ rad
   From **Eq 5-4** $A= D^2 (\pi - \sin(\pi)) /8 = D^2 \pi /8$ $P = \pi D/2$
   From **Eq 5-2,** $R_h = A/P = D/4$

   For a full pipe (h=D), $\alpha = 360° = 2\pi$ rad
   From **Eq 5-4** $A= D^2 (2\pi - \sin(2\pi)) /8 = D^2 \pi /4$ $P = 2\pi D/2 = \pi D$
   From **Eq 5-2,** $R_h = A/P = D^2 \pi / (4 \pi D) = D/4$

   The hydraulic radius for a half full pipe and a full pipe is the same !

3. **What is the flow going through a channel 6m wide, 1m deep with a slope of 0.0001 (n=0.015)?**

   $Q = A \cdot v = b \cdot h \cdot v$ wit b = 6 m and h = 1 m
   To determine v, we have to use Manning equation (**Eq 5-9)** where
   $R_h = A/P = b \cdot h / (b+2h)$ So $R_h = 6\times1/ (6+2\times1) = 0.75$ m
   S = 0.0001
   $C = (R_h^{1/6})/n$ with n =0.015 and $R_h = 0.75$ C = 63.55
   So $v = C\sqrt{R_h \cdot S}$ v = 0.55 m/s
   Then $Q = 6\times1\times0.55$ $Q = 3.3\ m^3/s$

4. **What is the depth of water in a channel 6m wide with a slope of 0.0001 and a flow of $6m^3/s$ (n=0.015) ?**

   Using Manning equation (**Eq 5-9)** $v = R_h^{2/3} \cdot S^{1/2} / n$
   with v=Q/A => $Q = A \cdot R_h^{2/3} \cdot S^{1/2} / n$
   with $A= b \cdot h=6h$
   with $R_h= (b \cdot h) / (b +2h) = 6h/(6+2h)=3h/(3+h)$
   $Q=b \cdot h \cdot (b \cdot h/(b+2h))^{2/3} \cdot (S)^{1/2} / n$

   Thus with b=6, we have $Q=6h \cdot (3h/(1+h))^{2/3} \cdot (0.0001)^{1/2} / 0.015$ and are looking to solve this equation to find the value of h for $Q=6\ m^3/s$, but it is too complicate to be solved.
   Therefore, trial iteration will be done to try to find it.
   With h=1 m depth, the flow is Q = 3.3 m^3/s, flow is too small thus h should be bigger.
   With h=2 m depth, the flow is Q = 9.03 m^3/s, already too high, thus the solution is in between.
   With h=1.5 m depth, the flow is Q = 6 m^3/s, the solution is found.

5. **What is the width of a rectangular channel to carry $13.5m^3/s$ with a 1.8m water depth and a slope of 0.0004 (n=0.012) ?**

   Here the exercise is similar as the previous one but we search the width of the channel with the new parameters for slope, n and depth, using the same equation as in the previous exercise:
   $Q=b \cdot h \cdot (b \cdot h/(b+2h))^{2/3} \cdot (S)^{1/2} / n$
   with the new values we have $Q=1.8b \cdot (1.8b/(b+3.6))^{2/3} \cdot (0.0004)^{1/2} / 0.012$

   With b=4 m width, the flow is Q = 11.6 m^3/s, thus to have a bigger flow is should be wider.
   With b=5 m width, the flow is Q = 15.5 m^3/s, the flow is too big, thus the solution is in between.
   With b=4.5 m width, the flow is Q = 13.5 m^3/s, the solution is found.

Answers to intermediary exercises

6. What is the minimum flow to have in a channel 6m wide and 1.5m water depth to be sure that the flow is turbulent?

For a turbulent flow Re > 10 000
From **Eq 5-1** $Re = 4\, R_h \cdot v / \nu$
The channel is 6 m wide and 1.5 m high
$Rh = b \cdot h / (b+2h) = 1$
So $v > Re \cdot \nu / (4R_h)$ as $Q = b \cdot h \cdot v$, $Q > Re \cdot \nu \cdot b \cdot h / (4R_h) = Re \cdot \nu (b+2h)/4 = 10^4 \times 10^{-6} \times 9 / 4$
To have a turbulent flow, Q must be superior to 0.0225 m3/s or 22.5 l/s

7. A cylindrical pipe with a slope of 0.002 should carry 2.30m^3/s, we want it to be 80% filled. What should be the diameter (to be done with the equations)?

If the pipe is 80 % filled, it means that the height h is equal to 80 % of the diameter D.
According to **Eq 5-4,**
$\alpha = 2 \text{ arcCos } (1- 2\ h/D) = 2 \text{ arcCos } (-0.6) = 4.43$

Using **Eq 5-2**, $R_h = A/P = D\ (\alpha - \sin \alpha) / 4\ \alpha$
Then as $Q = A \cdot v$, we can work by iterative approximation to solve
$8.Q\ n = (\alpha - \text{Sin } \alpha) \cdot D \cdot (D\ (\alpha - \sin \alpha) / 4\ \alpha)^{2/3} \cdot S^{1/2}$
As this equation cannot be solved easily, it should be done by iteration or by using the "Goal Seek…" function in Excel.
For Q = 2.3 m^3/s. S = 0.002, n= 0.0106 and α corresponding to 80% filling, the Diameter of the pipe should be : 1.24 m

8. What are the velocity and the height in a DN100 pipe with a flow of 20l/s and a slope of 40% (to be done with the fig. 5-1 & 5-2)?

Using fig 5-1, we can read for a DN100 at a slope of 40%, the flow of the full pipe: Q_f=40 l/s and the velocity is about v_f = 5.1 m/s
As we have a flow of 20 l/s, $Q_{\%} = Q / Q_f = 20 / 40 = 50\%$
Using fig 5-2, we can read $h_{\%} = 50$ thus $h = 100 \times 0.5 = 50$ mm
$v_{\%} = 100\%$ thus $v = v_{\%} \cdot v_f = 5.1$ m/s

9. What is the maximum acceptable slope for a smooth DN100 pipe with a flow of 10 l/s, if we want to limit the velocity at 3 m/s; at 5 m/s (to be done with the fig. 5-1 & 5-3)?

Using fig 5-1, we can read for a DN100 with a flow of 10l/s, the slope of the full pipe: S_f=2.5% and the velocity is about v_f = 1.25 m/s
As we want a maximum velocity of 3 m/s, $v_x = v_{max} / v_f = 3 / 1.25 = 2.4$
Using fig 5-3, we can read $S_x = 6$ thus $S_{max} = S_f \cdot S_x = 2.5 \times 6 = 15\%$
Using fig 5-3, we can read $h_{\%} = 44\%$ thus $h = 100 \times 0.44 = 44$ mm
The maximum admissible slope for this pipe, if we want to limit the water velocity at 3 m/s, is of 15% with a height of 44 mm.

For a limitation of 5 m/s the first step is the same, then we have:
As we want a maximum velocity of 5 m/s, $v_x = v_{max} / v_f = 5 / 1.25 = 4$
Using fig 5-3, we can read $S_x = 25$ thus $S_{max} = S_f \cdot S_x = 2.5 \times 25 = 62.5\%$
Using fig 5-3, we can read $h_{\%} = 30\%$ thus $h = 100 \times 0.3 = 30$ mm
The maximum admissible slope for this pipe, if we want to limit the water velocity at 5 m/s, is of 62.5% with a height of 30 mm.

10. What is the flow going through a V notch weir of 60° with the height of water of 10cm?

If we consider a turbulent flow, c=0.4 and h=0.1m, then
Using **Eq 5-10** , Q = 4 / 5 × 0.4 × tg(60°/2) × √(2 × 9.81) × $(0.1)^{5/2}$ = 0.00259 m^3/s = 2.59 l/s
The value is confirmed by the figure 5-4

The hypothesis on the flow can be confirmed by calculating the Re number
From **Eq 5-1** $Re = 4 \cdot Rh \cdot v / \nu$ $Rh = A/P$ and $Q = Av$ So $Re = 4Q / \nu P$
From trigonometry $P = 2h/\cos(\theta/2)$ thus $Re = 2 \times 0.00259 \times \cos(60/2) / 0.1 \times 10^6$
so Re = 44 000, it corresponds to a turbulent flow.

Answers to advanced exercises

11. Show that the best hydraulic section for a rectangular channel is h=b/2

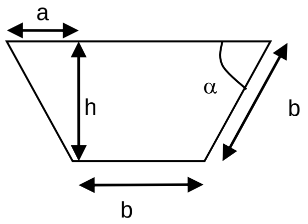
For a rectangular channel we have (eq 5-3) $A = b \cdot h$ (1) and $P = b + 2 \cdot h$ (2). It follows from (1) and (2) that $P = \frac{A}{h} + 2 \cdot h$ (3).

We want to minimize the wetted perimeter (P) for a given area (A). In other word, we have to derivate P as a function of h and set this to zero to find the minimum.

$\frac{\partial P}{\partial h} = -\frac{A}{h^2} + 2 = 0$ (4) This gives $\frac{A}{h^2} = 2$ (5) Substituting (1) into (5) we get $\frac{b \cdot h}{h^2} = 2$ (6) or $h = \frac{b}{2}$

12. Knowing that the best hydraulic section for a trapezoidal section is a half hexagon, calculate b, a and h for a trapezoidal channel having an area of 4m²?

Using eq 5-5 we know that $A = (b + a) \cdot h$ (1)



Given that our trapezoid is a half hexagon, we can deduct that α is an angle of 60° (the internal angles in a regular hexagon are of 120°). We also know that the bottom width (b) equals the side length

Therefore, using trigonometric formulas, $a = \cos(60°) \cdot b$ (2)

and $h = \sin(60°) \cdot b$ (3)

Substituting (2) and (3) into (1) we get

$A = (b + b \cdot \cos(60)) \cdot \sin(60) \cdot b$ (4) From there we find

$$b = \sqrt{\frac{A}{(\cos(60°) + 1) \cdot \sin(60°)}} = 1.754[m] \quad (5)$$

Then, from (2) and (5), we find

$a = \cos(60°) \cdot b = 0.877[m]$

and from (3) and (5), we find

$h = \sin(60) \cdot b = 1.520[m]$

13. What is the flow going through a rectangular weir of 2m width, with a crest height of 1 m and a height of water of 50cm?

We use eq 5-11 and 5-12

$Q = c \cdot L \cdot \sqrt{2 \cdot g} \cdot h^{5/2}$ (1)

where L = 2 [m]

h = 0.5 [m]

z = 1 [m]

$$c = 0.41 \cdot \left(1 + \frac{1}{1000 \cdot h + 1.6}\right) \cdot \left(1 + 0.5 \cdot \left(\frac{h}{h+z}\right)^2\right) = 0.41 \times \left(1 + \frac{1}{1000 \times 0.5 + 1.6}\right) \times \left(1 + 0.5 \cdot \left(\frac{0.5}{0.5+1}\right)^2\right)$$

$= 0.434$

So $Q = 0.434 \times 2 \cdot \sqrt{2 \times 9.81} \times (0.5)^{5/2} = 0.679 \left[m^3 / s\right]$

Answers to basic exercises

1. For a pump of 50 m³/h at 40 m head (NS≈20), what is the expected hydraulic power, pump efficiency, mechanical power, motor efficiency and power factor, active and total electrical power?

Hydraulic power, $P_{hydro}=\rho \cdot g \cdot h \cdot Q$

water density ρ=1 000 kg/m³

gravitational constant g=9.81 m/s²

manometric head h=40m

flow $Q=\frac{50m^3/h}{3600s/h}=0.0139m^3/s$

So, P_{hydro}=1000×9.81×40×0.0139=5 450 W=5.45kW

Pump efficiency η_p: with figure 6-2, (efficiency of centrifugal pumps), for a N_S of 20, and a flow of 50 m³/h (yellow line), we get an efficiency of 73%

Mechanical power: $P_{mec}=P_{hydro}/\eta_{pump}$=5 450/0.73=7 466 W=74.7 kW

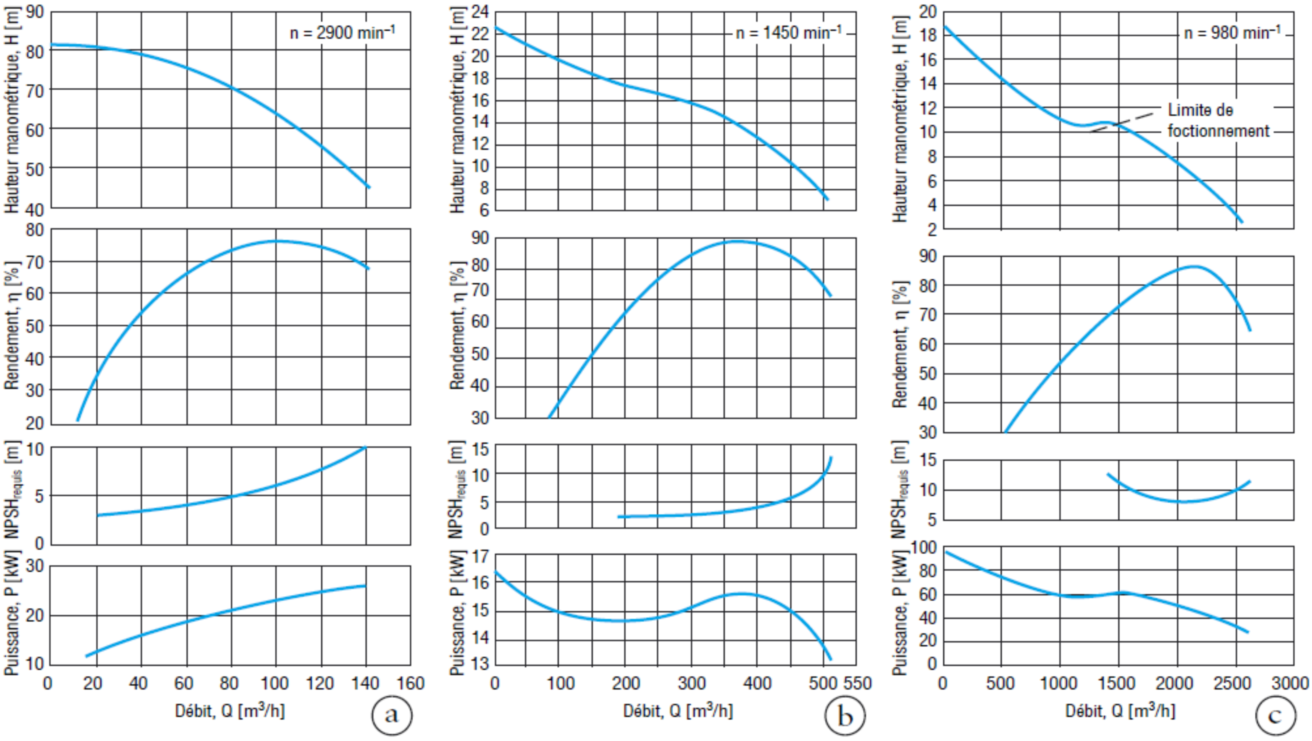
Motor efficiency η_m: with figure 6-1 (motor efficiency). For a rated power of approximately 75 kW and a 2-pole motor, we get a motor efficiency of 0.93

Power factor: with the same figure, but the pink line, we get a motor factor of 0.895

Total power: $S=\frac{\rho \cdot g \cdot h \cdot Q}{PF \cdot \eta_m \cdot \eta_p}=\frac{1000\times 9.81\times 40\times 0.0139}{0.895\times 0.93\times 0.73}=8974\,W=8.97kW$

Active power: P_{elec}=S·PF=8973.7×0.895=8032 W = 8 kW

2. What are the nominal head, flow, efficiency, NPSH and power for the pumps a, b, and c?



The nominal head and flow are the one corresponding to the highest efficiency. Therefore

	Pump a	Pump b	Pump c
Head, h	63 m	14 m	6.5 m
Flow, Q	100 m^3/h	375 m^3/h	2 200 m^3/s
Efficiency, η	76 %	89%	86%
NPSH	6 m	3 m	7m
Power, P	23 kW	15.5 kW	48 kW

3. What are the expected minimum and maximum flow for the pumps a, b, and c?

The minimum flow corresponds to the functioning limit point (when the HQ diagram makes a kind of bump). For the maximum flow, it is approximately when the lines on the charts end.

	Pump a	Pump b	Pump c
Minimum flow Q_{min}	20 m^3/h	200 m^3/h	1500 m^3/h
Maximum flow Q_{max}	140 m^3/h	500 m^3/h	2500 m^3/h

4. For the following system, what will be the duty point with the pump a, what is the power consumption?

Elevation : 200 masl

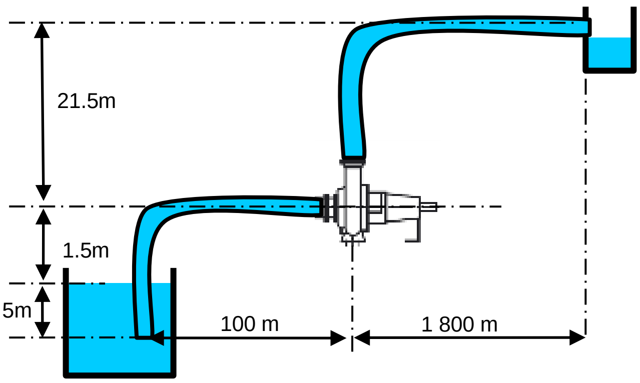
Temperature 20°c

All pipes are of new GI, DN 175

Total punctual friction losses:

- k_p = 15 for the suction part

- k_p= 5 for the delivery part



1) Estimate the flow

As the pump is already selected, we can use its nominal flow for the first estimation (i.e. Q_1 = 100 m^3/h)

2) Calculate h_{LP}

h_{LP}=punctual losses+linear losses= $\frac{v^2}{2g} \cdot (k_p + k_L)$ where

v=Q/A where $Q = \frac{100 m^3/h}{3600 s/h} = 0.027 m^3/s$ therefore for the first iteration

$$v = \frac{Q}{\frac{1}{4} \cdot \pi \cdot D^2} = \frac{0.027}{\frac{1}{4} \times \pi \times 0.175^2} = 1.15 m/s$$

k_p=15+5=20

k_L will depend on the velocity, but we can either use the chart in the annexes (head losses for pipe under pressure), or use equations 4-15 and 4-19.

$k_L = \lambda \cdot \frac{L}{D}$ where lamda is calculated using eq 4-19

To use eq 4.19, we need the Reynold's number

$$Re = \frac{v \cdot D}{\nu} = \frac{1.15 \times 0.175}{1 \times 10^{-6}} = 201\,102$$

which allows us to calculate λ=0.020

L is the length of all the pipes L=5+1.5+100+21.5+1800=1928 m

D is the diameter, 0.175m

$k_L = 0.020 \times \frac{1928}{0.175} = 218.5$ with equation 4-15 and 4-19

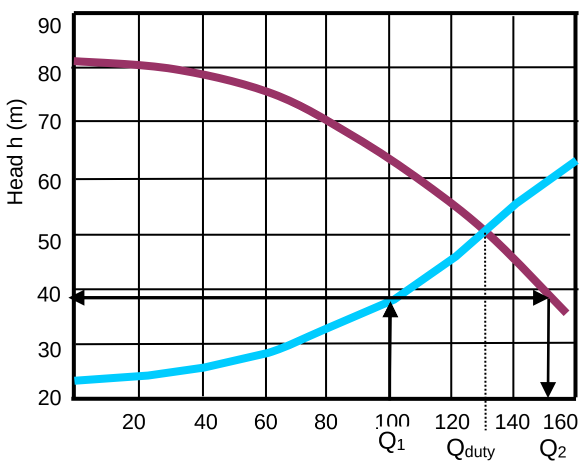
$$h_{LP} = \frac{v^2}{2g} \cdot (k_p + k_L) = \frac{1.15^2}{2 \times 9.81} \times (20 + 218.5) = 16.21\text{m}$$

3) Calculate the total head

$$h_{tot} = h_{static} + h_{losses} = (21.5 + 1.5) + 16.21\text{m} = 39.21\text{m}$$

4) Find a new Q (Q_2)

Looking back in the HQ chart, the flow corresponding to a head of 39.2m is 150 m^3/h. To find this value it is necessary to extrapolate the right hand-side of the chart by extending the end of the curve. This is shown in the following chart. The purple line is the QP curve given and the blue line is the network curve, which can be calculated as explained previously as a function of the flow



5) Iterate

For the next iteration, we will take the average between the flow with which we started our calculations (Q_1 100 m^3/h) and the flow we obtained at the end (Q_2 150 m^3/h) which gives us a flow of 125 m^3/h

The iteration are shown in the following table. At the end, we find a flow of 130 m^3/h.

it	Q1 m³/h	Q m³/s	v	Re	lamda	kL	stat head	linear loss	punctual loss	total loss	h tot	Q2 m³/h
0	100	0.028	1.15	202 101	0.019 83	218.5	23	14.85	1.36	16.21	39.21	150
1	125	0.035	1.44	252 626	0.019 52	215.1	23	22.85	2.12	24.97	47.97	135
2	130	0.036	1.50	262 732	0.019 48	214.6	23	24.65	2.30	26.95	49.95	130

In order to find the corresponding power consumption, we look in the QP chart and find a power consumption of 26kW

Answers to intermediary exercises

5. What is the number of poles and the slip value of the motor for the pumps a, b, and c?

For this question we use Eq 6-5 and the corresponding table. The number of rotation per minute (n) is written on the very top of each graph series for each pump. We then look in the table to find the frequency and number of poles to which it corresponds. The number n in the table might not correspond exactly to the number of the n indicated on the graph, as the slip value varies from one motor to another. After that, we have only one unknown in eq 6-5, which becomes solvable.

$n = 60 \cdot \frac{2 \cdot f}{nbPoles} - slip$ Therefore, $slip = 60 \cdot \frac{2 \cdot f}{nbPoles} - n$

a) The number n=2 900 corresponds to 50 Hz and 2 poles.

$$slip = 60 \times \frac{2 \times 50}{2} - 2900 = 100 RPM$$

b) The number n=1 450 corresponds to 50 Hz and 4 poles.

$$slip = 60 \times \frac{2 \times 50}{4} - 1450 = 50 RPM$$

c) The number n=980 corresponds to 50 Hz and 6 poles.

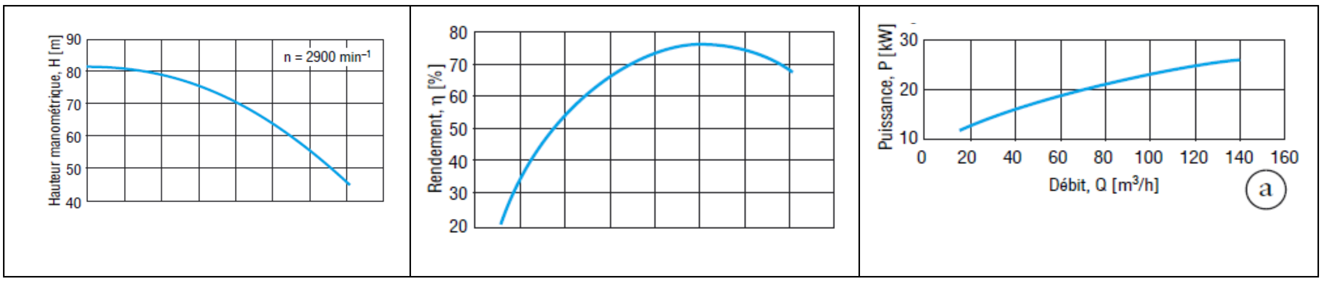
$$slip = 60 \times \frac{2 \times 50}{6} - 980 = 20 RPM$$

6. What is the specific speed for the pumps a, b, and c?

We use Eq 6-6, where Q and h are the nominal flows and heads found in 2.

a) $N_s = n \cdot \frac{Q^{0.5}}{h^{0.75}} = 2900 \times \frac{(100/3600)^{0.5}}{63^{0.75}} = 23$

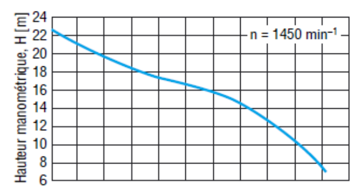
This N_s correspond to a high head impeller described in the section about specific speed



The HQ chart has a curve rather flat	The efficiency curve is rounded	The power consumption increases as the flow increases.

b) $N_s = n \cdot \frac{Q^{0.5}}{h^{0.75}} = 1450 \times \frac{(375/3600)^{0.5}}{14^{0.75}} = 65$

In this case, we have a low head impeller

<table>
<tr>
<td>
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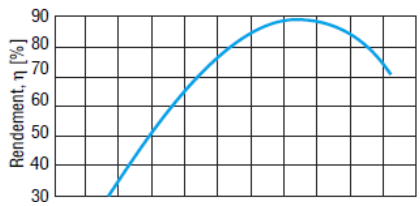
<td>
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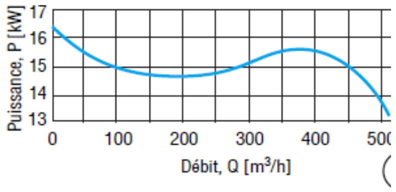
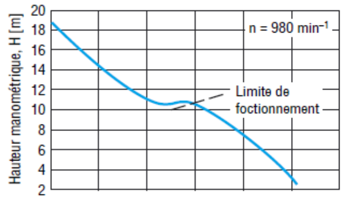
<td>
</td>
</tr>
<tr>
<td>The HQ chart has a steeper curve with a functioning limit point. Therefore, the functioning range of this pump is smaller than the one used in a</td>
<td>The efficiency curve is less rounded than the one in a (more pointed at the top)</td>
<td>The power consumption is not linear, but in overall decreases when the flow increases.</td>
</tr>
</table>

c) $N_s = n \cdot \frac{Q^{0.5}}{h^{0.75}} = 980 \times \frac{(2200/3600)^{0.5}}{6.5^{0.75}} = 188$

In this case, we have an axial flow impeller

<table>
<tr>
<td>
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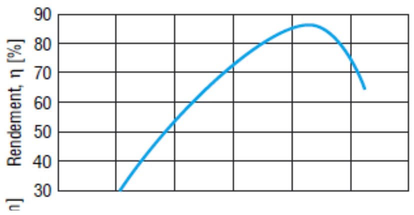
<td>
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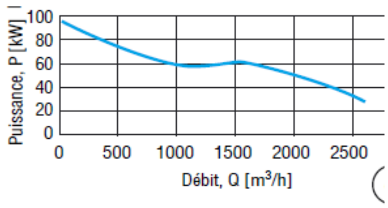
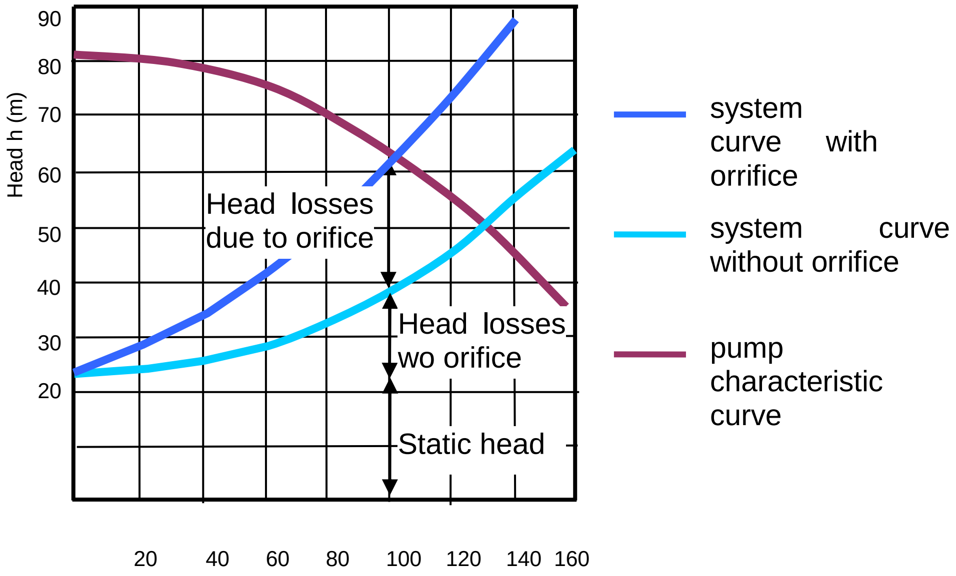
<td>
</td>
</tr>
<tr>
<td>The HQ chart is even steeper curve with a functioning limit point. The functioning range of this pump is very small for this kind of pumps</td>
<td>The efficiency curve is more pointed</td>
<td>The power consumption is not linear, but in overall decreases when the flow increases. The average slope of this line is steeper than the one in b</td>
</tr>
</table>

7. **If we want to adjust the working point of the pump used in exercise 4 to the nominal point with a throttling system, what should be the size of the orifice, what would be the power consumption?**

The best efficiency (nominal point) of this pump is for a flow of 100 m^3/h and a head of 65m. Given that for a flow of 100 m^3/h, the head losses are 16.2 m (first iteration of exercise 4), and the static head stays at 23m, the head losses that the orifice will have to create are:

$h_{p\text{-}orifice}$=63m-23m-16.2m=23.8m



With eq 4-14, we have

$h_{\text{p-orifice}} = k_{p-o} \cdot \frac{v^2}{2g}$ where v is the velocity corresponding to a flow of $100m^3/h$.

$v = \frac{Q}{\frac{1}{4} \cdot \pi \cdot D^2} = \frac{100/3600}{\frac{1}{4} \times \pi \times 0.175^2} = 1.155$ m/s (again, this velocity can be read from the first iteration, exercise 4)

Therefore, $k_{p-o} = \frac{h_{\text{p-orifice}} \cdot 2g}{v^2} = \frac{23.8 \times 2 \times 9.81}{1.155^2} = 350$

Therefore, if we choose a sharp orifice, we find (with the "losses due to orifice" chart in chapter 4) a d/D ratio of 0.29. Given that our D=175mm, d=0.29×175=51mm (which is the diameter of the orifice, the edge of the orifice being sharp).

The power consumption will then be (looking in HP chart) of 23kW.

This corresponds to a reduction of 13% compared to the system as it is working in exercise 4.

8. With the same situation as exercise 4, what would be the new flow and power consumption, if we decrease the rotation speed, so that it reaches 80% of its initial value?

The first thing is to define a new HQ curve. For this, we calculate the flow and the head at minimal, maximal and optimal points

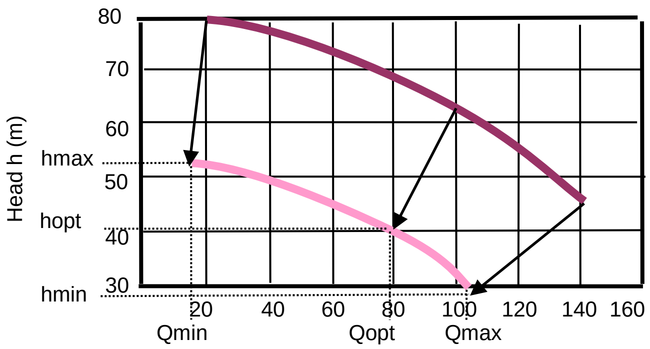
$$Q_2 = Q_1 \cdot \frac{n_2}{n_1} = 0.8Q_1$$

$$h_2 = h_1 \cdot \left(\frac{n_2}{n_1}\right)^2 = 0.64h_1$$

Therefore,

	100 %	80 %		100 %	80 %
Q_{min}	20 m^3/h	16 m^3/h	h_{max}	80 m	52 m
Q_{opt}	100 m^3/h	80 m^3/h	h_{opt}	63 m	40 m
Q_{max}	140 m^3/h	112 m^3/h	h_{min}	45 m	29 m

The HQ curve will look like the one given but shifted to the bottom and right. Knowing that, we can draw the HQ curve of the pump. It looks like that:



Next, we do the same as with point 4 with this new HQ curve.

1) <u>Estimate the flow</u>

We take the nominal flow, i.e Q_1=80 m^3/h

2) <u>Calculate h_{LP}</u>

H_{LP}=punctual losses+linear losses=$\frac{v^2}{2g}\cdot\left(k_p + k_L\right)$ where

$$v = \frac{Q}{\frac{1}{4}\cdot\pi\cdot D^2} = \frac{80/3600}{\frac{1}{4}\times\pi\times 0.175^2} = 0.92\text{m/s}$$

k_p=15+5=20

$k_L = \lambda\cdot\frac{L}{D}$ where lamda is calculated using eq 4-19

$Re = \frac{v\cdot D}{\nu} = \frac{0.92\times 0.175}{1\times 10^{-6}} = 161\,681$, which allows us to calculate λ=0.020

L is (as for point 4) 1928 m and D is 0.175m

$k_L = 0.020\times\frac{1928}{0.175} = 222.5$ with equation 4-15 and 4-19

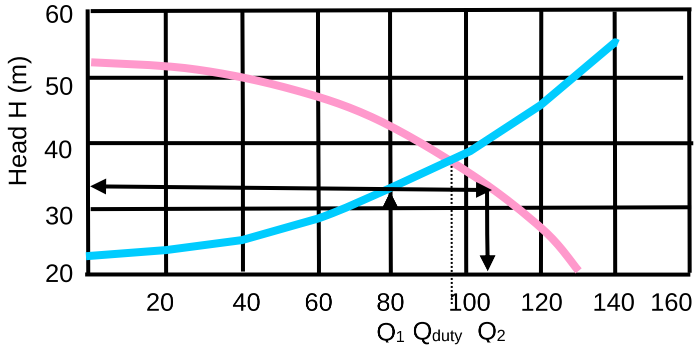
$$h_{LP} = \frac{v^2}{2g}\cdot(k_p + k_L) = \frac{0.92^2}{2\times 9.81}\times\left(20 + 222.5\right) = 10.55\text{m}$$

3) <u>Calculate the total head</u>

$$h_{tot} = h_{static} + h_{losses} = (21.5 + 1.5) + 10.5\text{m} = 33.5\text{m}$$

4) <u>Find a new Q (Q_2)</u>

Looking back at the pump characteristics, the flow corresponding to a head of 33.5m is 105m^3/h. This is shown in the following chart. As before, the purple line is the QP curve given and the blue line is the network characteristics, which is the same as the one drawn in point 4



5) <u>Iterate</u>

The average between Q_1 (80 m^3/h) and the flow we obtained at the end Q_2 (105 m^3/h) which gives us a flow of 92.5 m^3/h

The iteration are shown in the following table. At the end, we find a flow of 95 m^3/h.

it	Q1 m³/h	Q m³/s	v	Re	lamda	kL	stat head	linear loss	Punctual loss	total loss	h tot	Q2 m³/h
0	80	0.022	0.92	161681.2	0.020	222.50	23	9.68	0.87	10.55	33.55	105
1	92.5	0.026	1.07	186943.9	0.020	219.84	23	12.79	1.16	13.95	36.95	97
2	94.75	0.026	1.09	191491.2	0.020	219.42	23	13.39	1.22	14.61	37.61	96

For the power consumption, knowing that the flow is 95 m^3/h and that the power consumption for this flow is 22 kW, we will have $P_2 = P_1 \cdot \left(\frac{n_2}{n_1}\right)^3 = 22 \times (0.8)^3 = 11.3\text{kW}$

As it can be noticed, the power consumption passes from 26 kW without adjustment, to 23kW with a throttling system and to 11.3 kW with a speed reduction system. Compared with a throttling system (where we have almost the same flow reduction), the power consumption with a speed decrease system is much more efficient.

9. If the system is working as per exercise 4, what is the maximum suction height? And what is the maximum suction height if it is working as in exercise 7?

The maximum suction head is when it is equal to the height of the water column minus the head losses in the suction part and the NPSH required by the pump

$$h_{suc} = \frac{P_a - P_v}{\rho \cdot g} - h_{LP\text{-}S} - NPSH_r$$

At 200 masl, the atmospheric pressure is 98 000 Pa (chart chapter 2 page 3). In this exercise, we want to avoid cavitation. Therefore, we have to take the minimum pressure on the chart (light blue line), representing the smallest water column. The vapour pressure for water at 20°C is 2340 Pa (figure 2-3).

- The height of the water column is: $\frac{P_a - P_v}{\rho \cdot g} = \frac{98000 - 2340}{1000 \times 9.81} = 9.75m$
- The head losses in the suction part are calculated as follow

$h_{LP} = \frac{v^2}{2g} \cdot (k_p + k_L)$ where v=Q/A and k_p=15 (punctual losses in the suction part)

- The $NPSH_r$ is given in the chart NPSH versus flow.

a) When the system is working as per exercise 4

The flow for this system is 130 m^3/h

- The height of the water column: is 9.75m
- The head losses in the suction part: we have

  - k_p for the suction part that is 15

  - $k_L = \lambda \cdot \frac{L}{D} = 0.019 \times \frac{106.5}{0.175} = 11.85$ where lambda is from the last iteration of point 6,

L=5+1.5+100=106.5 and D is the diameter, 0.175m

$$h_{LP} = \frac{v^2}{2g} \cdot (k_p + k_L) = \frac{1.5^2}{2 \times 9.81} \times (15 + 11.85) = 3.08m$$

- The NPSH: For a flow of 130 m^3/h, we find a NPSH of 8 m

Therefore, the maximum is $h_{suc} = 9.75 - 3.08 - 8 = -1.33m$ this negative number indicates clearly that the system designed in point 6 only works if the pump is placed at least 1.35m below the water level. If not, cavitation will occur, will damage the pump and reduce the flow.

b) When the system is working as per exercise 7 (adjustment with a throttling system)

- The height of the water column: stays at 9.75m
- The head losses in the suction part: we have

  - k_p = 15 for the suction part

  - $k_L = \lambda \cdot \frac{L}{D} = 0.020 \times \frac{106.5}{0.175} = 12.07$ where lamda was recalculated for this flow with eq 4.19

$$h_{LP} = \frac{v^2}{2g} \cdot (k_p + k_L) = \frac{1.17^2}{2 \times 9.81} \times (15 + 12.07) = 1.89m$$

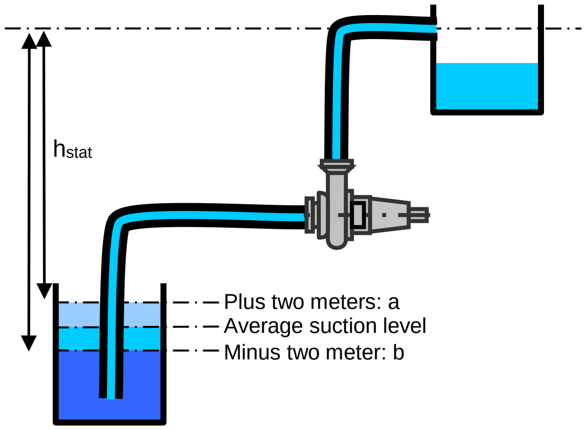
- The NPSH: For a flow of 100 m^3/h, we find a NPSH of 6 m

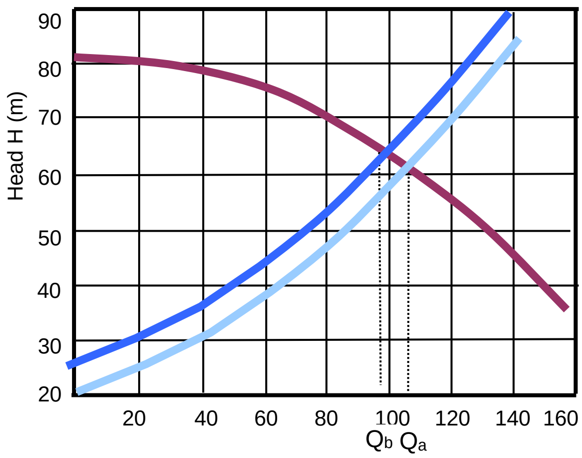
Therefore, the maximum is $h_{suc} = 9.75 - 1.89 - 6 = 1.86m$. This value being higher than 1.5 m (which is the suction height of the pump as it is designed in point 6) indicates that cavitation will not occur with this system

The maximum suction height between point a and b is increased because the flow is decreased, which will decrease the head losses and the NPSH required by the pump. Thus if we want this system to work without cavitation, the flow has to be reduced, by throttling, trimming or speed reduction.

Answers to advanced exercises

10. For the system used in exercise 7 (with a throttling system), knowing that the intake water level varies of plus or minus 2 meters, what would be the max and min flow, what would be the max suction height?





a) With plus 2 meters

- The flow

We have to do the same iterations as it was done in exercise 4. This time, the static head is 21m instead of 23m. We can begin the iteration at 100 m^3/h, since the flow will be practically the same (the static head does not change much).

The punctual losses coefficient is equal to the punctual losses in the suction part and delivery part plus the one of the orifice k_p=15+5+350=370

it	Q m^3/h	Q m^3/s	V m/s	Re	lamba	k_L	H static	Linear losses	Punctual losses	Total loss	h tot	Q2 m^3/h
0	100	0.028	1.15	202101.5	0.020	218.51	21	14.85	25.15	40.01	61.01	104.8
1	102.4	0.028	1.18	206928.0	0.020	218.13	21	15.54	26.37	41.91	62.91	99.8
2	101.1	0.028	1.17	204319.9	0.020	218.33	21	15.17	25.71	40.88	61.88	102.5
3	101.8	0.028	1.18	205773.5	0.020	218.22	21	15.38	26.07	41.45	62.45	101.0

Thus, by decreasing the static head, the flow is increased of about 1% : Q= 101 m^3/h.

- The maximum suction height (for this water level) is calculated as in exercise 9
  - the height of the water column is 9.75 m
  - the head losses in the suction part are

$$h_{LP} = \frac{v^2}{2g} \cdot \left(k_p + \lambda \cdot \frac{L}{D}\right) = \frac{1.18^2}{2 \times 9.81} \times (15 + 0.020 \times \frac{106.5}{0.175}) = 1.92\,\text{m}$$

  - the NPSH is around 6m for a flow of 101 m^3/h

Therefore the maximum suction height is 9.75m-1.92m-6m=1.84

It must be noticed that it is 1.84m above the water level for which we calculated the flow, which is 2 meters above the average suction level. Therefore, when the water level is 2m above the average water level, the pump should be placed at a maximum of 3.84m above the average water level.

b) with minus 2 meters

- The flow

We have to do the same iterations as it was done before. This time, the static head is 25m instead of 23m. Again, we start at a flow of 100 m^3/h.

it	Q m^3/h	Q m^3/s	v	Re	lamda	kL	stat head	linear loss	punctual loss	total loss	h tot	Q2
0	100.0	0.028	1.15	202101.5	0.020	218.51	25	14.85	25.15	40.01	65.01	94.1
1	97.0	0.027	1.12	196089.1	0.020	219.02	25	14.02	23.68	37.69	62.69	100.4
2	98.7	0.027	1.14	199489.0	0.020	218.73	25	14.49	24.51	38.99	63.99	96.9
3	97.8	0.027	1.13	197640.7	0.020	218.88	25	14.23	24.05	38.28	63.28	98.8
4	98.3	0.027	1.14	198668.4	0.020	218.80	25	14.37	24.30	38.68	63.68	97.7
5	98.0	0.027	1.13	198103.9	0.020	218.84	25	14.29	24.17	38.46	63.46	98.3

(The number of iterations necessary to converge on the result has increased, because the system curve is steeper, which makes the iteration process less efficient).

This time, by increasing the static head, the flow is decreased of about 2% : Q= 98 m^3/h.

- The maximum suction height (for this water level) is calculated as before
  - the height of the water column is 9.75 m
  - the head losses in the suction part are

$$h_{LP} = \frac{v^2}{2g} \cdot \left(k_p + \lambda \cdot \frac{L}{D} \right) = \frac{1.13^2}{2 \times 9.81} \times (15 + 0.020 \times \frac{106.5}{0.175}) = 1.78m$$

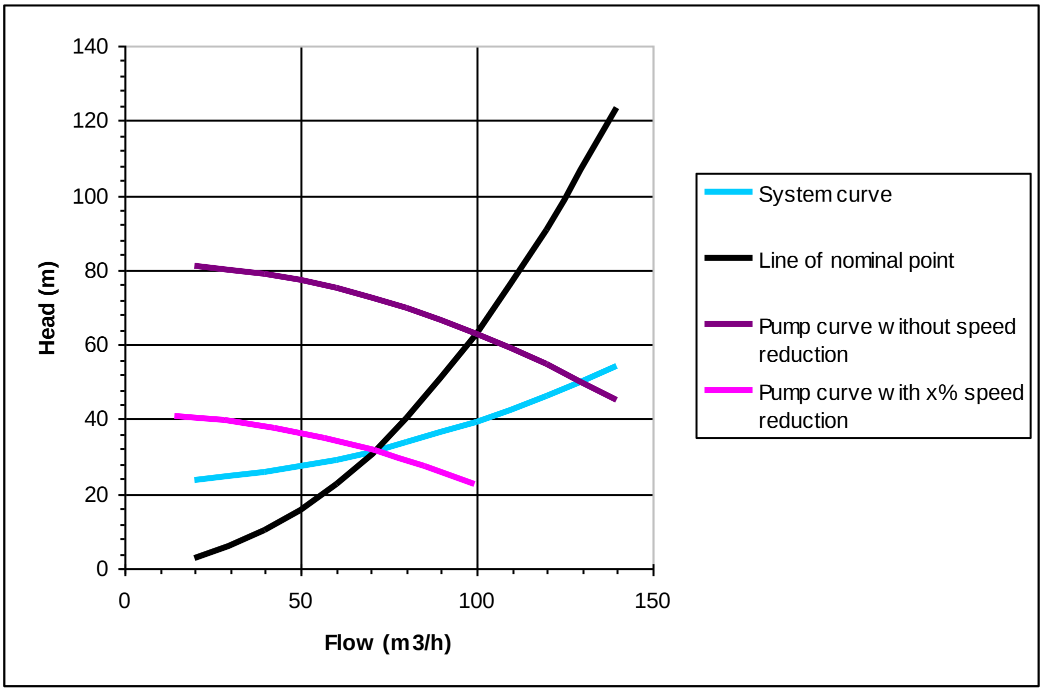
  - the NPSH is around 6m for a flow of 98 m^3/h

Therefore the maximum suction height is 9.75m-1.78m-6m=1.97m

This height corresponds to a height of 1.97m above the water level for which we calculated the flow (which is 2 meters below the average suction level). Therefore, the pump should be placed just below the average water level or lower (2m-1.97m=0.03m=3cm). This level is (not surprisingly) lower than the one obtained in a. Therefore, this is the value to be considered when doing the design to find the elevation of the pump avoiding cavitation. As a result, if the pump is place 1.5 m above the surface level (as it is the case in exercise 7), it will cavitate when the water level will drop down. As it can be seen, the variation of the water level in the intake tank has a big impact on cavitation, and should be taken into account when doing calculations.

11. With a frequency inverter, what should be the percent speed reduction to adjust the system to the nominal point of the pump? What is the corresponding flow?

As it is explained in the section about speed reduction, the line of nominal points with a speed reduction is parable. In order to find the point where the system is at its nominal point, we will have to find the place where the nominal point line crosses the system curve. This is shown on the following graph.



First, we have to find the equation of this parable. A parable is from the type

$y=ax^2+bx+c$ In our case, it is: $h_n=aQ_n^2+bQ_n+c$ (h_n and Q_n indicates the nominal head and nominal flow respectively).

We know that it passes by the origin, thus, c=0. Therefore, the equation of the parable will be from the type

$h_n=aQ_n^2+bQ_n$

To find the coefficient a and b, we need two points on this curve.

If we do not change the system, the nominal point is Q_n=100 m^3/h and h_n=63, which will be the first of the two needed points.

The second point (which is whichever point on the curve with r% rotation speed) will have apostrophe (H_n' and Q_n') to make the difference between the first (with 100% speed) and second point (r% speed).

$$(1)\ h_n = a \cdot Q_n^2 + b \cdot Q_n$$

$$(2)\ h_n' = a \cdot Q_n'^2 + b \cdot Q_n'$$

$$(3)\ Q_n' = Q_n \cdot r$$

$$(4)\ h_n' = h_n \cdot r^2$$

Substituting (3) and (4) into (2) we get

$$(5)\ h_n \cdot r^2 = aQ_n^2 \cdot r^2 + bQ_n \cdot r$$

which simplifies to (dividing by r)

$$(6)\ h_n \cdot r = aQ_n^2 \cdot r + bQ_n$$

From (1) we know that

$$(7)\ b \cdot Q_n = h_n - a \cdot Q_n^2$$

Putting (7) back into (6) we get

$$(8)\ h_n \cdot r = a \cdot Q_n^2 \cdot r + h_n - a \cdot Q_n^2$$

which can be written as

$$(9)\ h_n \cdot (r-1) = a \cdot Q_n^2 \cdot (r-1)$$

If follows that

$(10)\ a = \frac{h_n}{Q_n^2}$ In that case, given that Q_n=100 m^3/h and h_n=63, we have $a = \frac{63}{100^2} = 0.0063$

To find the coefficient b, we use (10) and (1) and find

$$(11)\ h_n = \frac{h_n}{Q_n^2} \cdot Q_n^2 + b \cdot Q_n$$

which gives us b=0

Therefore, our equation of the line of nominal point is $h = 0.0063Q^2$

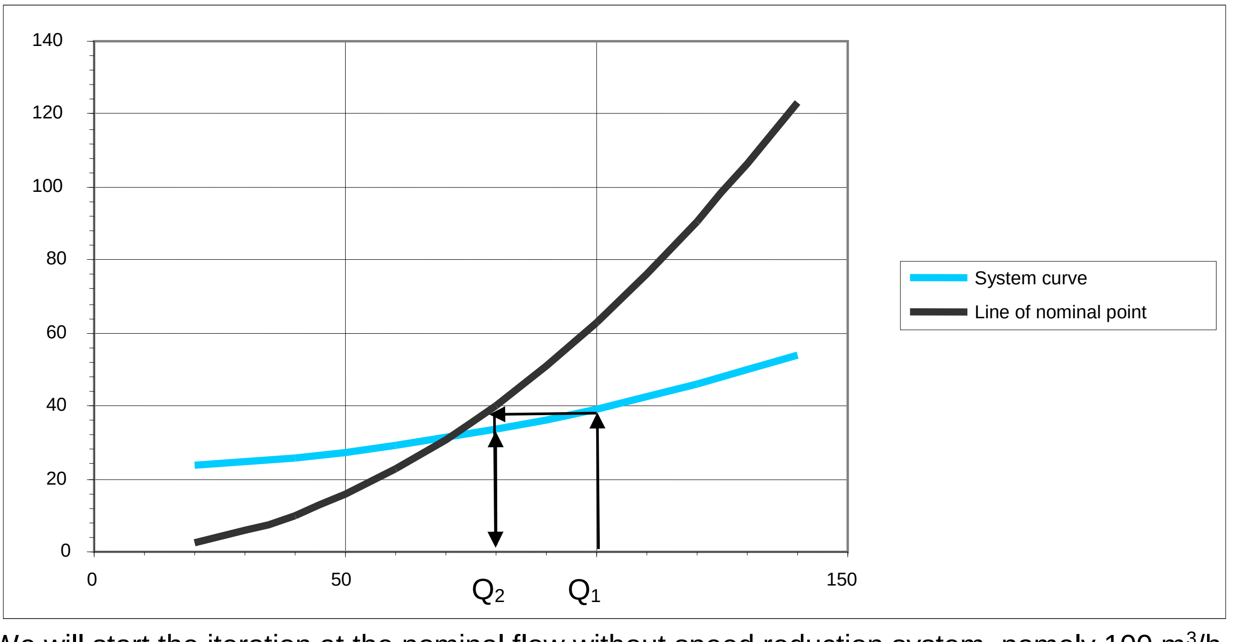
To find the intersection of the line of nominal point and the system curve, we will have to iterate in a slightly different way as before.

For a given flow Q_1, we will estimate the losses and the total head (static head plus head losses) as it was done before.

Then, for this head, we will find the corresponding flow Q_2 on the line of nominal points. To find the nominal flow for a given head, we solve the following equation for Q

$0.0063Q^2 - h = 0$ which gives us $Q = \sqrt{\frac{h}{0.0063}}$

This time, Q_2 can directly be taken for the next iteration (no need to do the average between Q_1 and Q_2). This is because between Q_1 and Q_2, we do not cross the intersection, as it was the case for the previous iterations (this can be seen in the following figure). Therefore, we get closer to the results every time and Q_2 can be taken for the next iteration.



We will start the iteration at the nominal flow without speed reduction system, namely 100 m^3/h. The iterations are shown in the following table.

it	Q m3/h	Q m3/s	v	Re	lamda	kL	stat head	linear loss	punctual loss	total loss	total h	Q2
0	100	0.028	1.15	202102	0.02	218.51	23	14.8538	1.3595	16.2134	39.21	78.9
1	78.89	0.022	0.91	159447	0.02	222.77	23	9.42562	0.8462	10.2718	33.27	72.7
2	72.67	0.02	0.84	146872	0.02	224.41	23	8.05645	0.718	8.77446	31.77	71
3	71.02	0.02	0.82	143529	0.02	224.89	23	7.71023	0.6857	8.39593	31.4	70.6
4	70.59	0.02	0.82	142671	0.02	225.01	23	7.62262	0.6775	8.30015	31.3	70.5

At the end, we get approximately a flow of 70 m^3/h.

Given that $\frac{Q_2}{Q_1} = \frac{n_2}{n_1}$ and that we have $\frac{Q_2}{Q_1} = \frac{70}{100} = 0.7$ it follows that $\frac{n_2}{n_1} = 0.7$

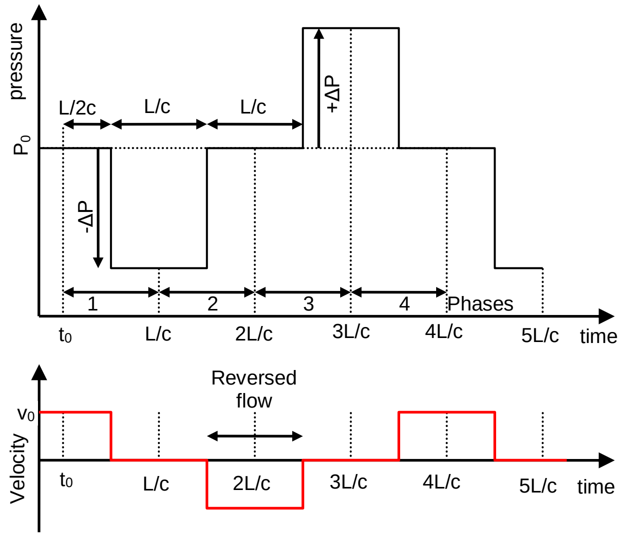
Therefore, the speed reduction should be of 70%

For the pump a we have a frequency of 50 Hz. Given that the rotation speed is directly proportional to the frequency, the frequency will have to be reduced to 70% of its initial value, which is 35 Hz.

Answers to basic exercises

1. **Draw the pressure and the velocity versus the time (L/c) at the middle of the pipe for an instantaneous valve closure, neglecting head losses.**

The surge will arrive at the middle of the pipe after half of the time needed to travel the pipe: L/2c and come back after two times this time : 2L/2c=L/c



2. **What are the velocity of the pressure wave (c), the head surge (Δh) and the return time in a cast iron pipe DN200 of 3 kilometres for a instantaneous decrease of velocity of 1.5 m/s?**

With the table of Cast iron pipe in the annexe N, we find with DN200 the value of c=1'216 m/s. To find Δh we have to multiple the given value by the actual velocity Δh=124×1.5=186 m. To find the return time we have to multiple the given value by the actual length in km Tr=1.6×3=4.8 s.

In this case the pressure surge is huge (almost 18 bar) but the return time very short, thus if it is possible to extend the closure or stopping time to 10 s the surge would be divided by 2 (9 bar) and with 20 s by 4 (i.e 4.5 bar).

3. **What are the velocity of the pressure wave (c), the head surge (Δh) and the return time in a PE pipe SDR11 of 5 kilometres for a instantaneous decrease of velocity of 2 m/s?**

With the table of PE pipe in the annexe N, we find with SDR11 (the diameter of the pipe is not needed, the water hammer will be the same for the all series) the value of c=342 m/s. To find Δh we have to multiple the given value by the actual velocity Δh=35×2=70 m. To find the return time we have to multiple the given value by the actual length in km Tr=5.8×5=29 s.

In this case, we can see that the pressure surge is smaller (~7 bar) but that the return time is quite important, almost half minute before the surge is back.

4. **What are the velocity of the pressure wave (c), the head surge (Δh) and the return time in a PVC pipe SDR17, OD 200 of 2 kilometres for a instantaneous closure with an initial velocity of 1 m/s?**

With the table of PVC pipe OD>100 in the annexe N, we find with SDR17 (the water hammer will be the same for the all series with OD>100) the value of c=490 m/s. The Δh is the one in the table as the velocity is of 1 m/s: Δh=50 m. To find the return time we have to multiple the given value by the actual length in km Tr=4.1×2=8.2 s.

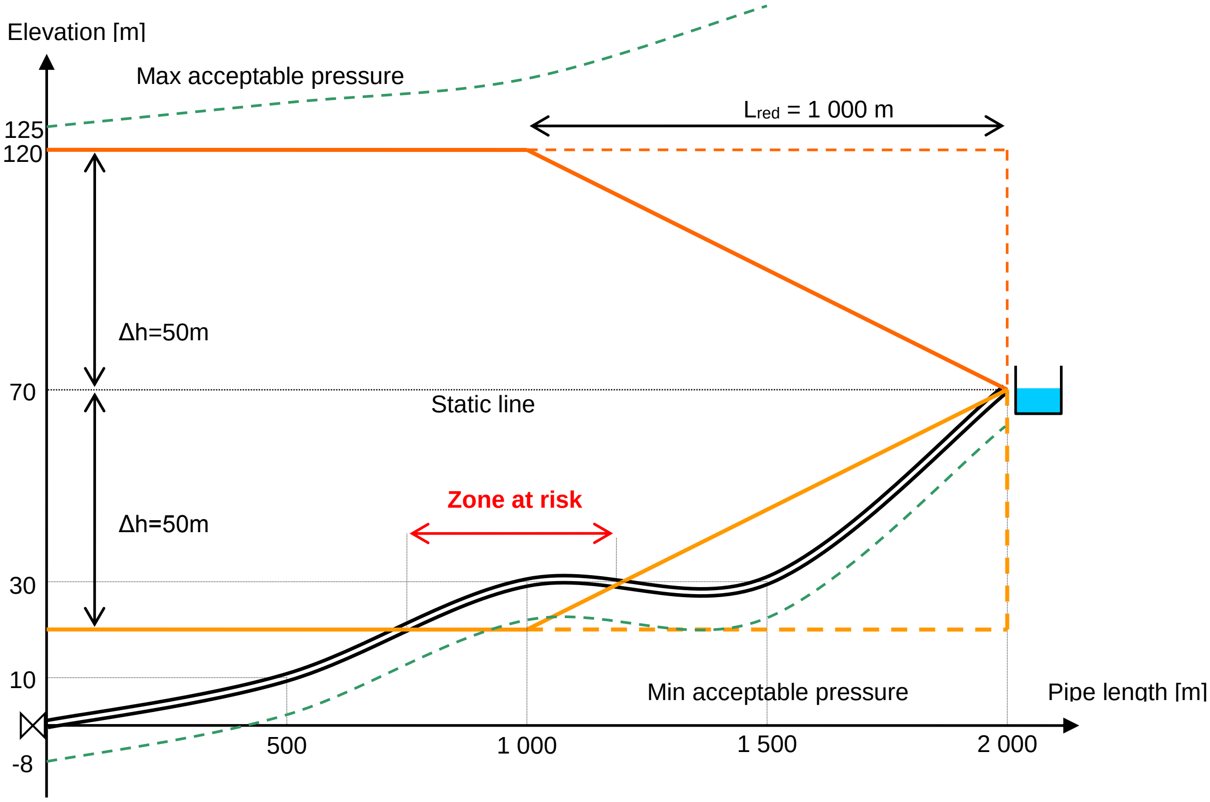
In this case, we can see that the pressure surge is quite small (~5 bar) and the return time not too long (less than 10 s). Thus this pipe should be not too much at risk.

5. What length of the previous pipe will have a reduced surge if the closure time is of 4.1 second?

The length with reduce surge is according to eq 7.4 is : $L_{red} = T_c\ c / 2 = 4.1 \times 490 / 2 = 1'000$ m. Thus with a decreasing time of half the return time, half of the pipeline will have reduced pressure durge.

6. Neglecting losses draw the envelope scheme for the previous pipe, knowing that its profile is as per the attached table. Is the pipe safe (assuming a linear front wave)? If not, what can be done?

	Pump	Pt 1	Pt 2	Pt 3	Tank
Pipe length	0	500	1'000	1'500	2'000
Elevation	0	10	30	30	70



We can see that a part of the pipeline (between 750 and 1 200) will be exposed to depressure, and that around 1 000 m it might even face cavitation. There is no risk of pipe burst (over pressure) as the maximum pressure ($h_{stat} + \Delta h = 120$) is smaller than the pipe PN (~125 m).
In order to avoid this depressure in the pipe, the following could be done:
- Increase the stopping time
- Install air valves letting air come into the pipe in the zone at risk

7. For the previous system what should be the closure time to limit the Δh at 30m?

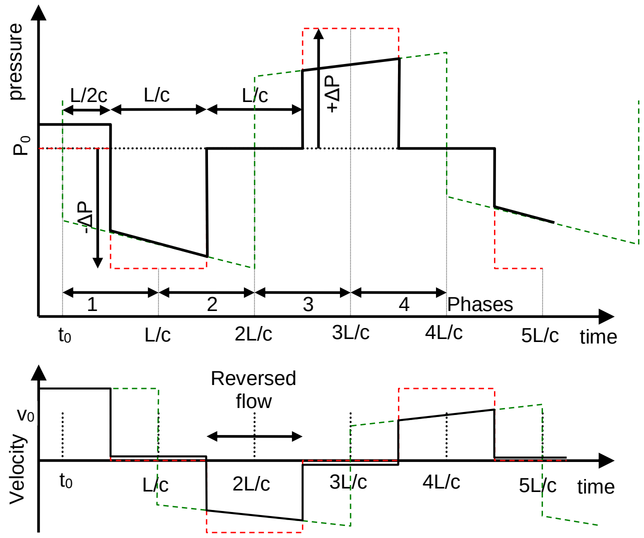
According to Eq. 7.5 $\Delta h_{max} = \frac{2L \cdot v_{in}}{g \cdot T_c} \rightarrow T_c = \frac{2L \cdot v_{in}}{g \cdot \Delta h_{max}} = \frac{2 \times 2000 \times 1}{9.81 \times 30} = 13.6s$

Thus, to reduce the Δh to 30 m, the closure time should be of 13.6 s. In theory, this would be also valid for a cast iron pipe or any material with a return time for this system smaller than 13.6 s.

Answers to intermediary exercises

8. **Draw the pressure and the velocity versus the time (L/c) at the middle of the pipe for an instantaneous valve closure, taking into consideration head losses.**

The surge will arrive at the middle of the pipe after half of the time needed to travel the pipe: L/2c and come back after two times this time : 2L/2c=L/c



9. **What are the velocity (c) the head surge (Δh) and the return time (T_r) of the pressure wave in a cast iron pipe DN200 (e=6.4 mm, K_{pipe}=140 GPa) of 3 km, if a pump of 170 m^3/h instantaneously stops working, with a temperature of water of 45°?**

We calculate c with Eq 7-1

$$c=\sqrt{\frac{K_{water}/\rho}{1+\frac{D}{e}\cdot\frac{K_{water}}{K_{pipe}}}}$$ where at 45°, K_{water} is 2.29 GPa and ρ=990.2 kg/m^3

The thickness e is 6.4 mm, K_{CI} is 140 GPa

The diameter D can be estimated as the nominal diameter (DN) so 200 mm

so $$c=\sqrt{\frac{K_{water}/\rho}{1+\frac{D}{e}\cdot\frac{K_{water}}{K_{pipe}}}}=\sqrt{\frac{2.29\times10^9/990.2}{1+\frac{200}{6.4}\cdot\frac{2.29\times10^9}{140\times10^9}}}=1237\text{ m/s}$$

Given that the flow is totally stopped, $\Delta v=v=\frac{Q}{A}=\frac{170/3600}{\frac{\pi}{4}\cdot\left(200\times10^{-3}\right)^2}=1.50\text{ m/s}$

The head surge is found with equation 7-1: Δh = Δv·c/g=1.5 × 1237 / 9.81 = 189.6 m

The return time is found with equation 7-3: T_r= 2L/c= 2 × 3000 / 1237 = 4.85 s

If we compare these results with the exercise 2 we can see that they are very close, the difference of taking in to consideration the water temperature of 45° make a difference of 1.5% on the head surge and of 1% on the return time.

10. For the previous pipe, we want to limit the head surge at 100m by having a longer stopping time, what should be in this case T_C?

According to equation 7-5 we have $T_c = \frac{2L \cdot v_{in}}{g \cdot \Delta h_{max}} = \frac{2 \times 3000 \times 1.5}{9.81 \times 100} = 9.17\,s$

BY increasing significantly the closure time, we will have almost half of the flow when the head surge is coming back therefore the Δv is divided by two, dividing thus also the head surge by two.

11. What are the velocity (c) the head surge (Δh) and the return time (T_r) of the pressure wave in a PE80 pipe PN12.5, OD200 (K_{PE}=0.7 GPa, e=18.2mm) of 5 km, if a pump of 150 m^3/h instantaneously stops working, with a temperature of water of 20°?

We calculate c with Eq 7-1

$c = \sqrt{\frac{K_{water}/\rho}{1 + \frac{D}{e} \cdot \frac{K_{water}}{K_{pipe}}}}$ where at 20°, K_{water} is 2.2 GPa and ρ=998.2 kg/m^3

The nominal thickness for this pipe is given 18.2 mm (an average of 19.2 could be used)

The bulk modulus K_{PE} is given as 0.7 GPa

The internal diameter D is thus per Annexe B 163.6 mm

so $c = \sqrt{\frac{K_{water}/\rho}{1 + \frac{D}{e} \cdot \frac{K_{water}}{K_{pipe}}}} = \sqrt{\frac{2.2 \times 10^9 / 998.2}{1 + \frac{163.6}{18.2} \cdot \frac{2.2 \times 10^9}{0.7 \times 10^9}}} = 274\,m/s$

Given that the flow is totally stopped, $\Delta v = v = \frac{Q}{A} = \frac{150/3600}{\frac{\pi}{4} \cdot \left(200 \times 10^{-3}\right)^2} = 1.982\,m/s$

The head surge is found with equation 7-1: Δh = Δv·c/g=1.982 × 274 / 9.81 = 55.5 m

The return time is found with equation 7-3: T_r= 2L/c= 2 × 5000 / 274 = 36.4 s

This time if we compare these results with the exercise 3 we can see that they are not so close, as the bulk modulus given is quite different from the one used in the table in annexe N.

12. What are the friction coefficient (K_{LP}) and the head losses (h_{LP}) of the previous pipe with a roughness of 0.07 mm, neglecting the punctual losses? What is the attenuation of the negative head surge due to the losses at the tank?

With equation 4-9 we find Re = D · v / *v* = 163.6 × 1.982 / 1.01 × 10^{-6} = 322 000

With equation 4-19 or fig 4-5 we find λ = 0.0147

As the singular losses are neglected, we find with equations 4-15

K_{LP} = λ · L / D = 0.0147 × 5 000 / 0.1636 = 449

h_{LP} = v^2 · λ / 2g = 1.982^2 × 449 / (2 × 9.81) = 89.9 m

Thus, the attenuation of the negative head surge at the tank is equal to h_{losses}/2, about 45m. In this situation where the losses are quite high as the velocity is important, we can see that the remaining negative head surge at the tank is only of 10m, thus the importance to take them in to account.

13. What are the attenuation of head surge at the valve ($h_{v\text{-}att}$), the remaining velocity at the tank (v_{t1}) and the attenuation of the positive head surge at the tank ($h_{t\text{-}att}$) of the previous pipe?

With equation 7-6 we find $h_{v\text{-att}} = \dfrac{\left(\sqrt{1+2\dfrac{k_{LP}v_{in}}{c}}-1\right)^2 c^2}{2gk_{LP}} = \dfrac{\left(\sqrt{1+2\dfrac{449\times 1.982}{274}}-1\right)^2 274^2}{2\times 9.81\times 449} = 25.8\,\text{m}$

With equation 4-9 we find :

$$v_{t1} = \frac{k_{LP}v_{in}^2}{4c} = \frac{449\times 1.982^2}{4\times 274} = 1.607\,\text{m/s}$$

$$h_{t\text{-att}} = \frac{2\left(\sqrt{1+\dfrac{k_{LP}(v_{in}-v_{t1})}{c}}-1\right)^2 c^2}{gk_{LP}} = \frac{2\left(\sqrt{1+\dfrac{449\times(1.982-1.607)}{274}}-1\right)^2 274^2}{9.81\times 449} = 2.5\,\text{m}$$

Thus when the surge arrive to the tank, the velocity in the depression zone reaches 1.607 m/s. This explains why the negative head surge is so small at the tanks as the difference of velocity is then only of 1.982-1.602=0.375 m/s.

The attenuation of the positive head surge at the valve is of 25.8 m thus the positive head surge at the valve is of 55.5 - 25.8 ≈ 30 m. NB this value as to be added at the static level, not at the dynamic one. This value is much smaller than the initial head losses (89.9 m) thus the positive head surge is not affecting the pipe at the valve.

The attenuation of the positive head surge at the tank is of 2.5 m thus the positive head surge at the tank is of 55.5 – 89.9 / 2 – 2.5 ≈ 8 m. This remaining positive head surge at the tank is very small.

See the next exercise correction for an illustration of these results.

14. For the previous pipe, draw the developed profile of the pipeline, the static and dynamic lines, and the envelope schemes knowing that its profile is as per the attached table. Is the pipe safe? If not, what can be done?

	Pump	Pt 1	Pt 2	Pt 3	Tank
Pipe length	0	1'000	1'000	1'000	2'000
Elevation	0	0	5	10	30

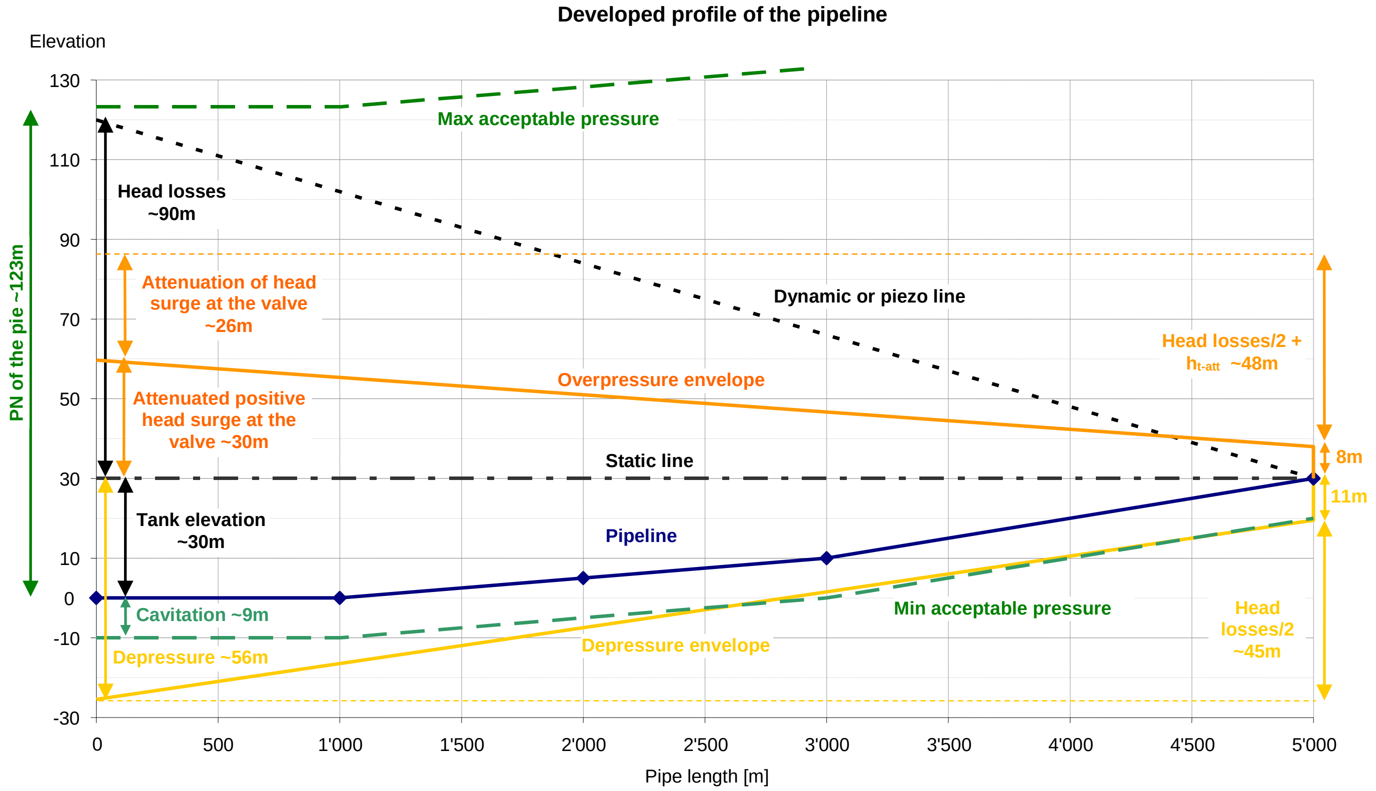
As explain in the section 7.6, the developed profile should be used but as the slopes are small it doesn't make a big difference if it is taken into consideration or not.

All the values calculated in the previous exercises were reported in this chart.

It can be seen that the superior envelope is much lower than the maximum acceptable pressure, thus there is no risk of pipe burst in case of water hammer.

However the inferior level is lower than the ground level, even lower than the cavitation line (minimum acceptable pressure), thus depressure and cavitation will happen in this pipeline if it is not protected.

Possible solutions might include a pump bypass system, or air valve. Nevertheless, into this situation the best would be to increase the diameter of the pipe, decreasing also the losses and necessary PN for the pipe, this will thus decrease the thickness and further the head surge.



15. What would be the effect on the head surge on the previous exercises of having a Young's modulus (K_{PE})of 1.1 GPa instead of 0.7 GPa?

The surge velocity will be at 341 m/s instead of 274 m/s increasing the head surge at 70 m instead of 56 m. This will significantly increase the problematic of the depressure at the valve, even for bigger diameters. This shows the importance to have a good information from the supplier about the properties of the pipes and in case of doubt to take a good margin between the envelope and the maximum or minimum tolerable pressure.

Answers to advanced exercises

16. For exercise 2-4, what would be the change in ID (DN for metallic pipe) following the change in pressure due to water hammer?

In order to calculate ΔD, we use a formula from chapter 1 (Eq 1-5)

$$\Delta D = D \cdot \frac{\Delta P}{K} \cdot \frac{D}{2e}$$

K is the Young's modulus of the pipe material.
For the first exercise, to find ΔD in mm, we have

$$\Delta D = 200 \times \frac{1053400}{125 \times 10^9} \times \frac{200}{2 \times 6.4} = 0.026\text{mm}$$

Therefore, when the pipe will be in overpressure, the DN will be
D_o=200+0.026=200.026 mm (the pipe will be expanded)
When the pipe will be in under pressure, the DN will be
D_u= 200-0.026=199.97.

The same procedure is done for exercise 3 & 4. The results are shown it the following table

Ex	ΔP Pa	DN or ID mm	e mm	K GPa	ΔD mm	D_o mm	D_u mm
2							
3							
4							

We can draw the following conclusion

- even if the ΔP is bigger for cast iron, the change in diameter will be less than for PE (because PE is easily distorted).
- even if the ΔP is bigger for exercise 4 than for 3, the change in pipe's diameter will be bigger for 3, because the wall thickness is smaller (which makes the pipe more easily distorted)

It can be noticed that the expansion due to changes in pressure in the pipe is not significant, because even for PE pipe with a bigger expansion, it is around half a millimetre added to the outside diameter, which is less than the tolerance margin for PE pipe. For example, for a PE pipe having an OD of 200 mm, the maximum outside diameter will be of 201.2 mm (which is bigger than 200.5 obtained with pressure expansion).

These calculations were done for expansion due to pressure changes only because of water hammer (thus ignoring the initial pressure in the pipe). This means that the diameter can be more expanded because the total change in pressure will be bigger. Furthermore, if we have an initial pressure, it must be checked that the PN of the pipe can bear the total pressure (initial pressure +ΔP).