DESIGN OF WATER SUPPLY SYSTEM

PART 1
BASICS OF HYDRAULICS

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GENEVA 2014

REFERENCE

Table of content

*** Sections to be done for the basic level**

Used symbols and units

Quantity	Unit		Description
A	m^2		Surface area in square meter
b	m		Width in meter
c	m/s		Water hammer surge velocity
D	m		Diameter in meter
E	mWC		Energy in meter of water column
e	mm		Pipe thickness
f	N		Force in Newton ($kg \cdot m/s^2$)
g	m/s^2		Earth gravity assumed generally as 9,81 m/s^2
H	m		Elevation in meter above sea level (masl)
h	m		Height or depth in meter
I	A		Electrical intensity (amperes)
K	Pa		Bulk modulus in Pa or GPa
k_l	-		Coefficient of linear losses
k_p	-		Coefficient of punctual losses
l	m		Length in meter
m	kg		Mass in kilogram
n	- / rpm		Manning's factor / rotation speed
Ns	-		Specific speed of a pump
P	Pa		Pressure in Pascal (N/m^2) or bar (1 Bar= 100’000 Pa)
P	W		Power in watt / Electrical active power
Q	m^3/s		Flow in cubic meter per second / electrical reactive power
S	%		Slope in meter per 100m / Electrical apparent power
t	s,m,h		Time usually in second, but might be minute or hour
T	°C-K		Temperature in centigrade or Kelvin degree (0°K = -273,15 °C) / Torque
U	V		Electrical tension or voltage in volts
v	m/s		Velocity in meter per second
V	m^3		Volume in cubic meter
α (alpha)	°-rad		Angle in degree (360) or radian (2π)
φ (phi)	-		Electrical phase angle
η (eta)	-		Efficiency
λ (lambda)	-		Friction coefficient
µ (mu)	Pa·s		Dynamic viscosity
ν (nu)	m^2/s		Kinematic viscosity
π (pi)	-		Pi = 3.1415
ρ (rho)	kg/m^3		Density or specific mass in kilo per cubic meter
ω (omega)	rad/s		Rotation speed

Prefixes for units

da	deca	10^1	d	deci	10^{-1}
h	hecto	10^2	c	centi	10^{-2}
k	kilo	10^3	m	mili	10^{-3}
M	mega	10^6	µ	micro	10^{-6}
G	giga	10^9	n	nano	10^{-9}
T	tera	10^{12}	p	pico	10^{-12}

Introduction to Basics of Hydraulics

What you will learn
The hydraulics notions useful to design water supply system.

Why
Ensure a basic and common understanding of the necessary theory to design water supply system.

Duration of the training
15 to 30 hours

Generality about this course

This course is the first part of the Design of Water Supply System methodology.
It makes the review of the important parts of hydraulics necessary to design WSS.
It is aimed for engineers or technicians with good understanding of water system.

Nevertheless, as illustrated in the attached diagram, it does not yet explain how to design the main components of the WSS. This is done in the second part of the training with the explanation on how to use several Excel tools to assist the engineer in his design.

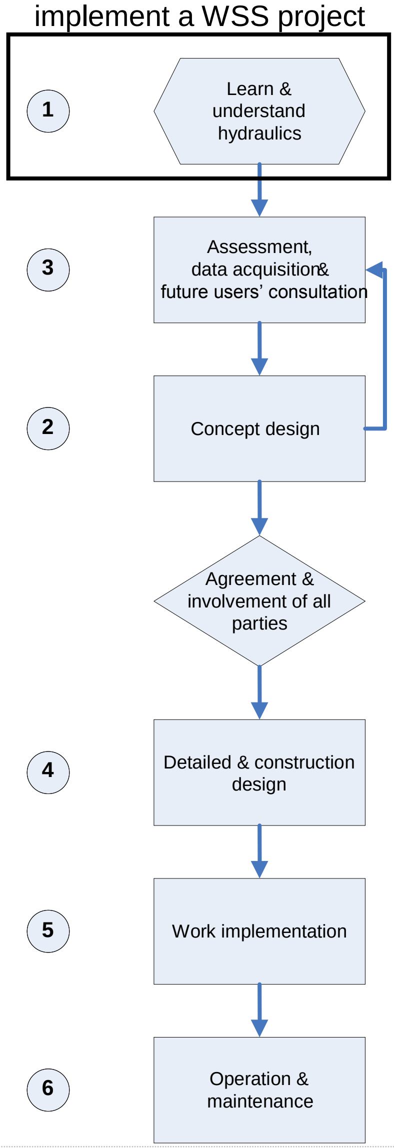
The course is divided in 7 chapters, for each of them you have a theoretical part to be study and some exercises to make sure that the theory was well understood. The corrections of the exercises are also given, but it is important to try to do the exercises without the solutions, and use them only if you are blocked.

It is also divided in **two levels**

The **Basic level** is the minimum requirement to have a based understanding on the main principles covering the main situations, without going too much in to complicate equations and special cases. The sub chapters to be done for this basic level are indicated by a star *.

The **Intermediate level** is the remaining theory, going more in details in the theory and covering most of the situation encountered (low/high temperature, high elevation, flatten pipes, ect).

Methodology to design & implement a WSS project



Chapter 1. Properties of fluids and pipes

What you will learn
Understand the basic essentials of fluid properties.

Why
It is important to understand and master the characteristic behaviours of fluids, especially of water and of air. This lays the groundwork for an understanding of hydrostatics and hydrodynamics - the following chapters of this course.

Duration of chapter 1
1 to 2 hours

1.1. Definition *

A **fluid** is any substance which flows because its particles are not rigidly attached to one another. This includes all liquids, all gases and even some materials which are normally considered as solids, such as glass. Liquids are almost incompressible while gases are readily compressible; liquids occupy a more or less constant volume and have a surface, whereas gases can expand and contract to fully occupy an available volume.
In hydraulics, water and air are the two main fluids of interest. Their main properties are described below.

1.2. Density *

The density of a material is defined as the mass per unit volume. The SI units for density are kg/m^3:

ρ: density in kilos per cubic meter $[kg/m^3]$
m: mass in kilos $[kg]$
V: volume in cubic metres $[m^3]$

$$\boxed{\rho = \frac{m}{V}} \quad \textbf{Eq. 1-1}$$

For a gas, the density will vary a lot with pressure and temperature:

P: absolute pressure $[N/m^2]$ or $[Pa]$
R: gas constant, for dry air it is about 29.3 $[m/K]$
T: temperature in Kelvin $[K]$ $(0\ °C = 273.15\ K)$
g: earth gravity (9.81 m/s^2)

$$\boxed{\rho = \frac{P}{R \cdot T \cdot g}} \quad \textbf{Eq. 1-2}$$

This is a variation of the ideal gas law:

P: is the absolute pressure $[Pa]$
V: volume in cubic meter $[m^3]$
T: is the temperature in Kelvin $[K]$

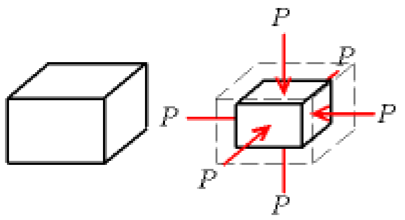
$$\boxed{\frac{P \cdot V}{T} = cte} \quad \textbf{Eq. 1-3}$$

The density of a liquid does not vary significantly with temperature; for example, the density of water varies by 0.3% between 5°C and 25°C (see annexe). In comparison, the density of a gas will vary by 7% over the same temperature range.

Relative density is defined as the ratio of the density of a substance to the density of a given reference material. The reference material is usually water. For example the relative density of mercury is 13.5 and for ethanol it is 0.789

1.3. Young's and bulk modulus

The Young's modulus for solids and the bulk modulus for fluids (K) measure the resistance to uniform compression. It is defined as the pressure increase needed to cause a given relative decrease in volume. Its base unit is the Pascal.



$$\Delta P = -K \frac{\Delta V}{V_0}$$ **Eq. 1-4**

Bulk modulus for some materials K	
Air	101 kPa (isotherm)
Water	2.0 - 2.3 GPa
Steel	160 - 200 GPa
Cast iron	80 - 170 GPa
Concrete	30 - 50 GPa
Glass	35 - 55 GPa
Asbestos	23 - 24 GPa
PE	0.7 - 1.2 GPa
PVC	3 - 4.7 GPa

The following formula can be used to calculate the change in diameter of a pipe due to an increase in internal pressure ΔP:

D: diameter of pipe [m]
e: thickness of pipe [m]
K: Bulk modulus of pipe [Pa]
ΔP: Pressure increase [Pa]

$$\frac{\Delta D}{D} = \frac{\Delta P}{K} \cdot \frac{D}{2 \cdot e}$$ **Eq. 1-5**

The compressibility of water and pipes can be usually neglected in hydraulics for steady flows; it is less than 1% for maximum permissible pressures - this effect is overshadowed by the tolerances used for pipe production.
However, this is not the case for a transitional situation (water hammer), where compressibility influences the speed and amplitude of a perturbation as it propagates through the pipes.

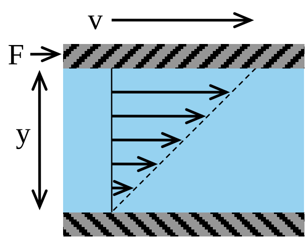
1.4. Viscosity

The viscosity of a fluid is a measure of its resistance to a tangential force; this resistance is mainly caused by interactions between the fluid's molecules.
Consider two large parallel plates, close to each other (y is small) and separated by a given fluid. For the upper plate to move at a constant velocity, a force F should be applied. This force will be proportional to the surface area of the plates and to the velocity of the upper plate, and is inversely proportional to the distance between the plates.

F: force in Newton [N]
μ: dynamic viscosity [Pa·s]
A: Surface of the plates [m2]
v: velocity of the upper plate [m/s]
y: distance between the plates [m]

$$F = \mu \cdot \frac{A \cdot v}{y}$$ **Eq. 1-6**

$$\mu = \frac{F}{A} \cdot \frac{y}{v}$$ **Eq. 1-7**



When the dynamic viscosity is independent of the shear stress (F/A), μ is constant (at a given temperature) and the fluid is called Newtonian, this is the case for water and most gases.

Kinematic viscosity ν, is a useful variable in hydraulics. It is defined as the ratio of dynamic viscosity to density.
It is expressed in [m^2/s] or in Stokes [St].

$$\nu = \frac{\mu}{\rho}$$ **Eq. 1-8**

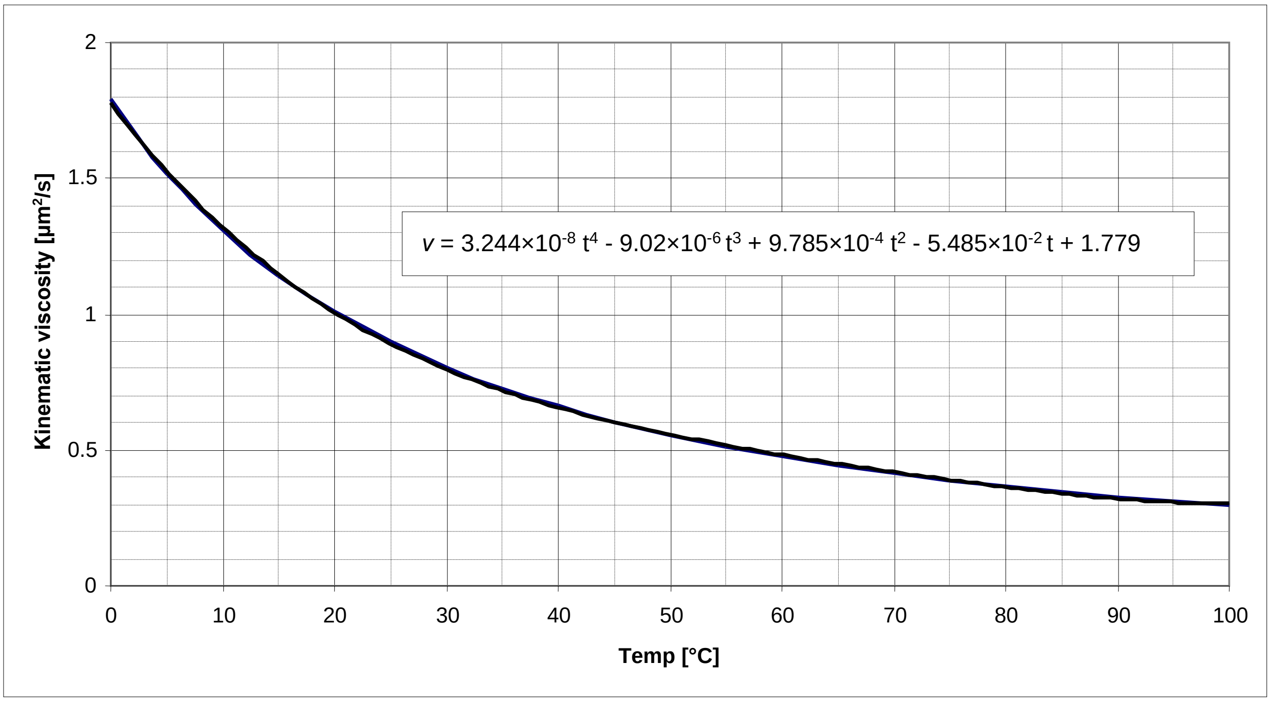


Fig 1- 1 Water viscosity at different temperatures

As shown in the chart, the viscosity decreases considerably as temperature increases. For instance, between 0°C and 20°C the viscosity of water decreases by 44%.

1.5. Phase transformations

Phase transformations refer to changes in the physical state of matter. Elements and simple compounds can generally exist as either solids, liquids or gases.
For water (H_2O), these states can be ice, liquid water and/or water vapour.
The state of a material at a given moment depends on its composition and on the temperature and the pressure exerted.

For instance at sea level (~ 10^5 Pa), water will be ice below 0°C, liquid between 0°C and 100°C, and water vapour above 100°C. At a pressure of 611 Pa, water will be solid ice up to 0.1 °C and will then sublimate directly to the vapour state.

These states are outlined for water in this phase diagram.

NB: For a better understanding, the scales are not respected.

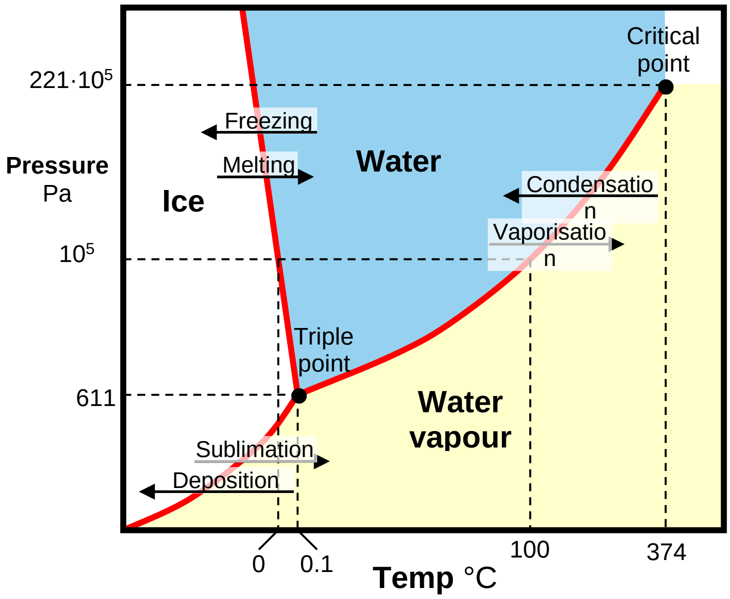


Fig 1- 2 Phase diagram of water

The triple point: The single combination of pressure and temperature at which liquid water, solid ice, and water vapour can coexist in a stable equilibrium occurs at exactly 273.16 K (0.01 °C) and a pressure of 611 Pa. At that point, it is possible to change all of the substance to ice, water, or vapour by making small changes in pressure or temperature.

***The critical point**: The vapour-liquid critical point denotes the conditions above which distinct liquid and gas phases do not exist. As the critical temperature is approached, the properties of the gas and liquid phases approach one another, resulting in only one phase at the critical point: a homogeneous supercritical fluid. The heat of vaporization is zero at and beyond this critical point, so there is no distinction between the two phases. Above the critical temperature a liquid cannot be formed by an increase in pressure, but with enough pressure a solid may be formed. The critical pressure is the vapour pressure at the critical temperature.*

Note that when there is a mix of elements (like air & water) the situation becomes more complicated (which is why we have water vapour in the air below 100°C). This will be further developed in the next chapter.

Table 1 : Properties of water

Temp [°c]	Density ρ [kg/m^3]	Viscosity ν [m^2/s]	Vapour Pressure [kPa]	Bulk Modulus [GPa]	Temp [°c]	Density ρ [kg/m^3]	Viscosity ν [m^2/s]	Vapour Pressure [kPa]	Bulk Modulus [GPa]
0	999.9	1.792×10^{-6}	0.61	2.04	50	988.1	0.556×10^{-6}	12.34	2.30
5	1000.0	1.519×10^{-6}	0.87	2.06	55	985.7	0.513×10^{-6}	15.75	2.31
10	999.7	1.308×10^{-6}	1.23	2.11	60	983.2	0.477×10^{-6}	19.93	2.28
15	999.1	1.141×10^{-6}	1.70	2.14	65	980.6	0.444×10^{-6}	25.02	2.26
20	998.2	1.007×10^{-6}	2.34	2.20	70	977.8	0.415×10^{-6}	31.18	2.25
25	997.1	0.897×10^{-6}	3.17	2.22	75	974.9	0.390×10^{-6}	38.56	2.23
30	995.7	0.804×10^{-6}	4.24	2.23	80	971.8	0.367×10^{-6}	47.37	2.21
35	994.1	0.727×10^{-6}	5.63	2.24	85	968.6	0.347×10^{-6}	57.82	2.17
40	992.2	0.661×10^{-6}	7.38	2.27	90	965.3	0.328×10^{-6}	70.12	2.16
45	990.2	0.605×10^{-6}	9.59	2.29	95	961.9	0.311×10^{-6}	84.53	2.11
50	988.1	0.556×10^{-6}	12.34	2.30	100	958.4	0.296×10^{-6}	101.33	2.07

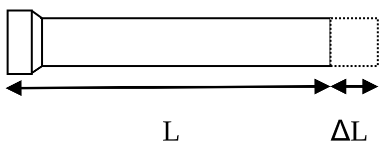
1.6. *Thermal expansion of pipes*

The change in temperature does not only influence the properties of water (like density and viscosity) but also has impacts on the pipes, which will expand as the temperature rises. For some materials, as for polyethylene, the dilatation effect can be non-negligible. With an increase in temperature, the pipe will increase its length as well as its diameter.

ΔL: total elongation of the pipe [mm]
L: length of the pipe before expansion [m]
ΔT: change in temperature [°K]
α_T: thermal expansion coefficient [mm/m°K]

$$\boxed{\Delta L = L \cdot \Delta T \cdot \alpha_T}$$

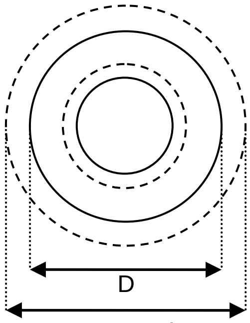
Eq. 1-9



ΔD: total increase in pipe's diameter [mm]
D: (inside or outside) diameter of the pipe before expansion [m]
ΔT: change in temperature [°K]
α_T: thermal expansion coefficient [mm/m°K]

$$\boxed{\Delta D = D \cdot \Delta T \cdot \alpha_T}$$

Eq. 1-10



Thermal expansion coefficient of some materials (mm/m°K)	
PE	0.200
PVC	0.080
Steel	0.011
Cast iron	0.012
Glass	0.009
Concrete	0.010

Therefore, the main consequences of thermal expansion from a hydraulic point of view will be an increase of the cross-sectional for plastic pipes, which can influence the flow. The increase in head loss due to the elongation of the pipe is negligible. In practice, the increase in length will mainly matters when we deal with hot fluids, but it is not the purpose of this course.

References for this chapter:

WEB:

http://www.thermexcel.com/english/ressourc/pdcgas.htm
http://en.wikipedia.org/wiki/Viscosity
http://en.wikipedia.org/wiki/Bulk_modulus

Books:

Série Schaum, Mécanique des fluides et hydraulique, Ranald V. Giles
ELBS, Fluid Mechanics 2nd edition, Douglas, Gasiorek, Swaffield

Basic exercises

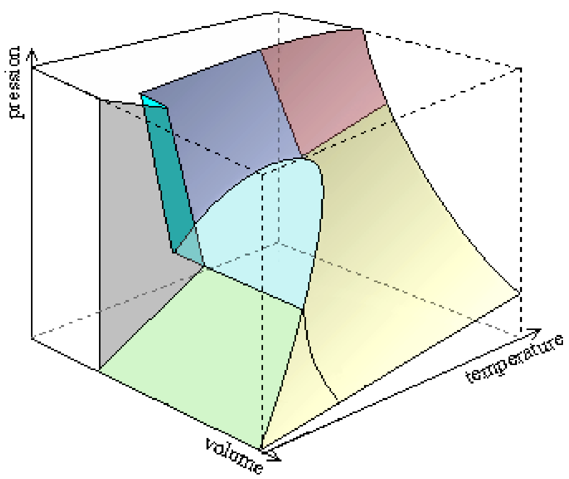
1. What is the mass of 1 dm^3 of mercury?
2. What is the density of sea water at 5°C knowing that the average salinity is 35 grams per litre?
3. What is the density of air at a pressure of 10^5 Pa and 30°C?
4. In a closed air tank we have a pressure of 10×10^5 Pa at 0°C. As the maximum pressure authorised for the vessel is 12×10^5 Pa, what is the maximum temperature admissible?
5. How does the density of water change with an increase in pressure (30×10^5 Pa) at constant temperature (5°C)?

Intermediary exercises

6. What corresponding reduction in the volume of a $1m^3$ steel cube is caused by an increase of 20×10^5 Pa?
7. In what state does water exist at 5×10^5 Pa and 0°C?
8. What will be the length and diameter of a PE pipe (original length 20m, original diameter 100mm), after thermal expansion due to temperature changing from 5°C to 25°C?
9. A plate 50 x 50 cm is supported by a water layer 1 mm thick. What force must be applied to this plate so that it reaches a speed of 2m/s at 5°C and at 40°C?

Advanced exercises

10. When a PE pipe with an outside diameter of 200 mm is pressurized to 10×10^5 Pa, what are the % increases in diameter for pipe wall thicknesses of 9.1 mm, 14.7 mm, and 22.4 mm?
11. On the following three dimensional (P,V,T) phase diagram for water, indicate the following areas, lines or points:
    - critical point (point) & triple point (line)
    - liquid water (area); solid ice (area); water vapour (area); supercritical fluid (area)
    - liquid water & solid ice (area); liquid water & water vapour (area); solid ice & water vapour (area)



Chapter 2. Hydrostatics

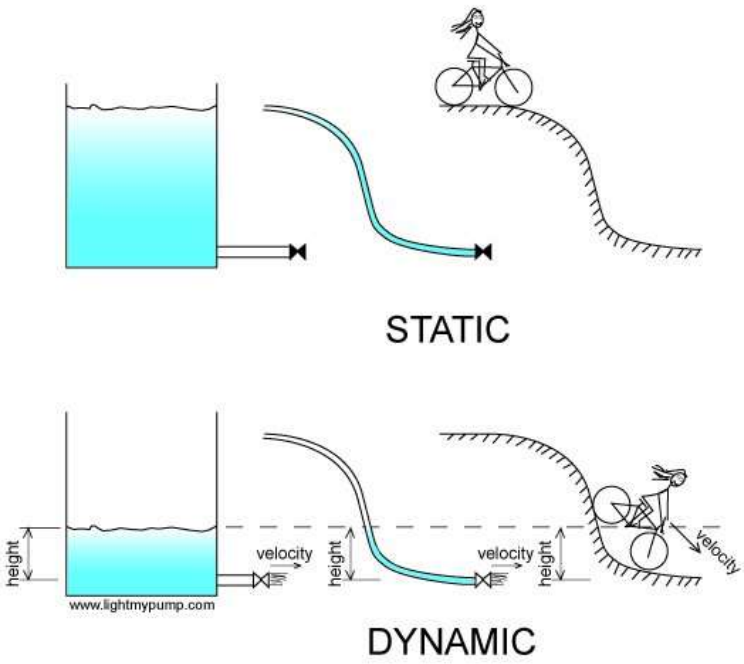
What you will learn
Understand and assimilate the principles of atmospheric pressure and water pressure in hydrostatics.

Why
A solid understanding of the pressure characteristics for systems involving water at rest is an essential requirement in order to understand hydrodynamics.

Duration of chapter 2
1 to 2 hours

*2.1. Introduction **

Hydrostatics is the study of the mechanical properties of fluids that are not in motion. In a water supply system, this is assumed to be the situation at night, when there is no one using the supply and leaks are insignificant/ignored. Water supply pipes are – and remain - under pressure.



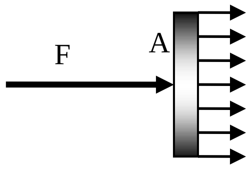
Hydrodynamics will take into account the movement of water through piping systems.

*2.2. Pressure **

The pressure is the force per unit area applied in a direction perpendicular to the surface of an object.

P : Pressure in Pascal [N/m^2]
F: Force in Newton [N]
A: Surface in square metres [m^2]

$$\boxed{P = \frac{F}{A}}$$ **Eq. 2-1**



Following International System of Units, a pressure of 1 Pascal is created by applying a force of 1 Newton (the force applied by 100g) acting on a plate of 1 square metre. If the same force is applied on a half square meter plate, the pressure will be doubled (2 Pa). Since a Pascal is quite a small unit, common multiples are more frequently used in practice, as given in the following table.
The pressure exerted by a liquid on a surface is directly proportional to its weight; therefore it cannot be negative. Similarly, the pressure exerted by a gas is a function of the impact of its particles against a surface (i.e. a function of its mass and temperature); therefore it cannot be negative.

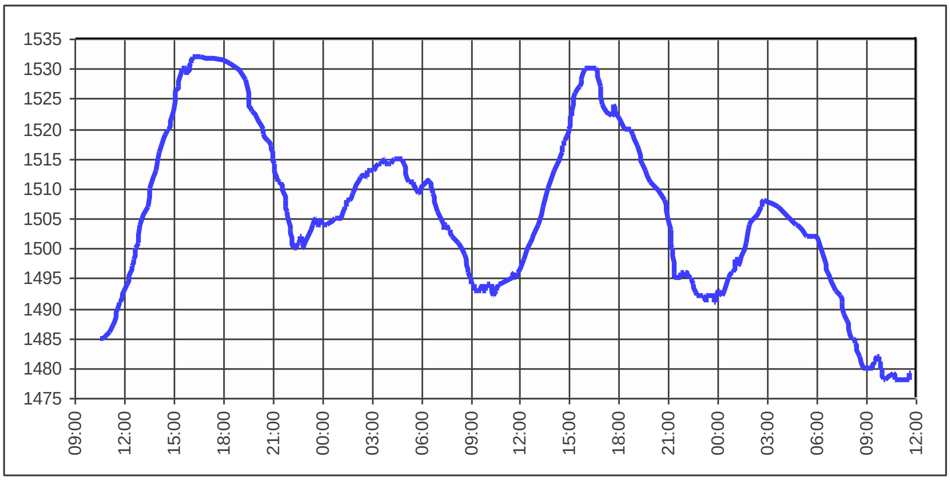
Different units used for pressure :

	Pascal (Pa)	**Bar (bar)**	**Atmosphere (atm)**	**torr (Torr)**	**Pound-force per square inch (psi)**
1 Pa	≡ 1 N/m^2	10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	100'000	≡ 10 N/cm^2	0.98692	750.06	14.5037744
1 atm	101'325	1.01325	≡ 1 atm	760	14.696
1 Torr	133.322	1.3332×10^{-3}	1.3158×10^{-3}	≈ 1 mmHg	19.337×10^{-3}
1 psi	6.894×10^{3}	68.948×10^{-3}	68.046×10^{-3}	51.715	≡ 1 lbf/in^2

The hecto Pascal (hPa) is the unit used in meteorology (1 hPa = 100 Pa)

2.3. Atmospheric pressure

Atmospheric pressure is the force exerted by the weight of air in the Earth's atmosphere. The average pressure at sea level is arbitrarily defined as 1 standard atmosphere (symbol: atm) which is equivalent to 101'325 Pa. An air column of ten square centimetres in cross-section, measured from sea level to the top of the atmosphere, weighs approximately 103.3 Kg.



Atmospheric pressure varies at a given point with the local conditions, (temperature of air, humidity, wind, moon and sun, etc).

In the tropics, significant diurnal pressure cycles are observed, this effect is known as atmospheric tide.

Fig 2- 1 Cyclic variations in atmospheric pressure over two days in Goma

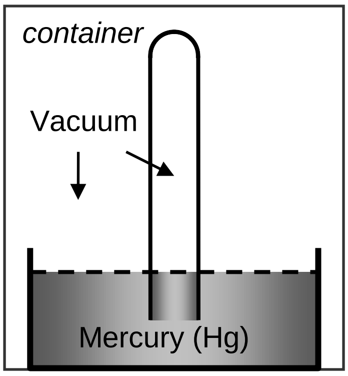


Fig 1

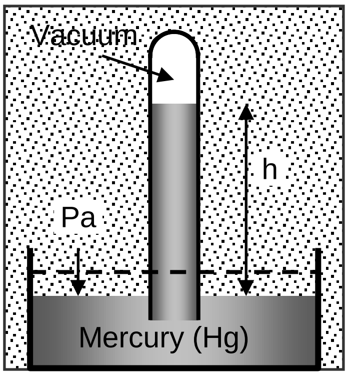


Fig 2

$$h = \frac{Pa}{\rho \cdot g}$$ **Eq. 2-2**

h: height of the liquid column
Pa : atmospheric pressure
ρ : density of the liquid
g : acceleration due to gravity
A : Section of the tube

When there is no gas (vacuum) in a container, there is no pressure. If we dip the bottom of a test tube in a bowl of mercury in a complete vacuum, the level of the mercury will remain at the same level inside the tube and in the bowl (fig 1). If air is then allowed to enter the container, it will push down on the surface of the mercury in the bowl and a column will rise inside the tube until the weight of this column counterbalances the air pressure.

To calculate the height of this column, the force due to the air pressure equals the force due to the weight of the mercury column:

$$F_{Hg} = m_{Hg} \cdot g = \rho_{Hg} \cdot V_{Hg} \cdot g = \rho_{Hg} \cdot h \cdot A \cdot g$$
$$F_a = P_a \cdot A$$
$$F_a = F_{Hg} \Rightarrow P_a = \rho \cdot h \cdot g$$

As elevation increases there is less atmospheric mass, so atmospheric pressure decreases with increasing elevation. A simplified relationship (not taking temperature differences into account) is shown graphically.

P : Mean atmospheric pressure [Pa]
h : Altitude [m]

$$P(h) = 101325\left(1 - \frac{0.0065 \cdot h}{288.15}\right)^{5.255}$$ **Eq. 2-3**

In this graph, we can see that at 1'000 m above sea level (masl), the mean atmospheric pressure is about 900 hPa (11% less than at sea level); and 750 hPa at 2'500 masl, corresponding to the altitudes in Asmara or Addis Ababa, (25% less than at sea level).

In such cases, it might be necessary to take atmospheric pressure into consideration for system design. Min and max pressure depend on location but can be estimated at a first stage at -41 to + 31 hPa from average pressure.

The P min has to be used for design, as it is the most critical case.

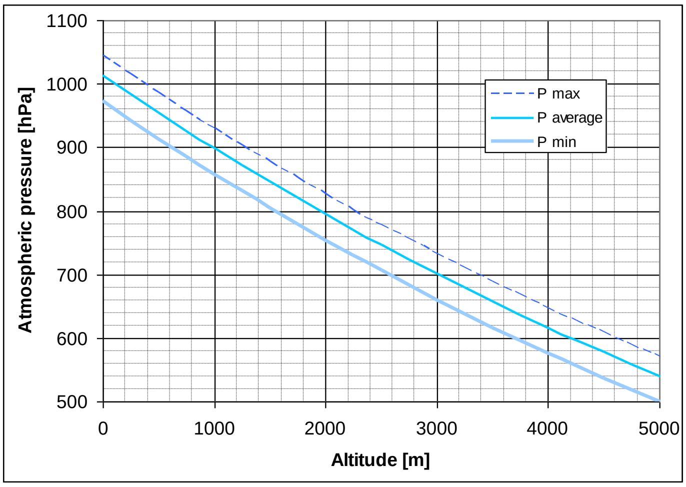


Fig 2- 2 Variation of pressure with altitude

2.4. Vapour pressure

The vapour pressure is the pressure at which water will vaporise (pass from liquid to vapour). For instance, at a fixed pressure of 1013 hPa (1atm), water will start to boil at 100 °C and it will remain at this temperature until it is completely vaporised.

At a constant 20°C, water remains liquid above 24 hPa. However, if the system pressure falls below this threshold, the water will start to vaporise, producing **cavitation** (constant temperature, decrease of pressure).

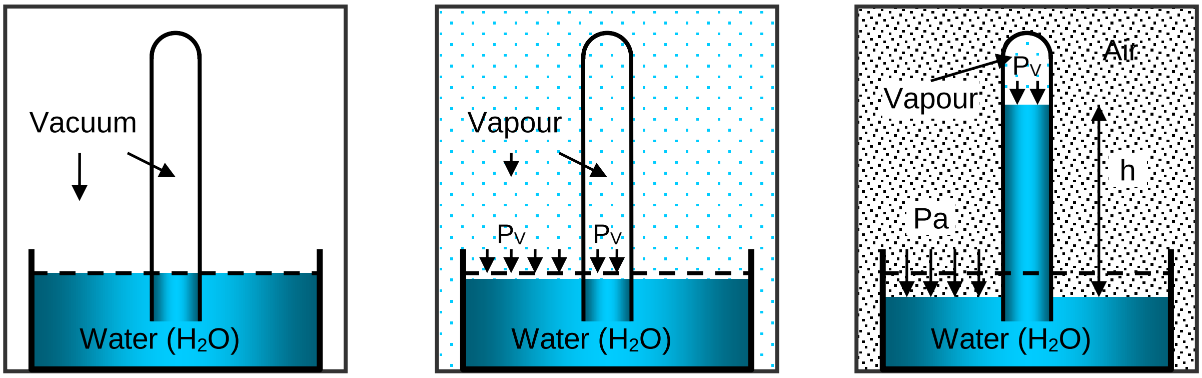


Fig 1 Fig 2 Fig 3

If water at 20°C is placed in a bowl in a container under vacuum (fig 1), as the pressure is very low, it will evaporate until the vapour pressure is reached in the container (fig 2). It will then be in a phase equilibrium with two phases, water vapour and liquid water at a pressure of 24 hPa.
If the temperature rises, more water will evaporate and the pressure will rise. If the temperature decreases, the vapour will condense and the pressure will decrease.
If air at atmospheric pressure is allowed to enter the container (fig 3), a water column will rise in the tube until the column weight balances the pressure differences. While the P_V in the container will be included (not added) in the Pa, the Pv in the tube will remain.

In this case, on top of the weight of the water column, you have the vapour pressure, balancing the atmospheric pressure, therefore the Eq 2-2 becomes:

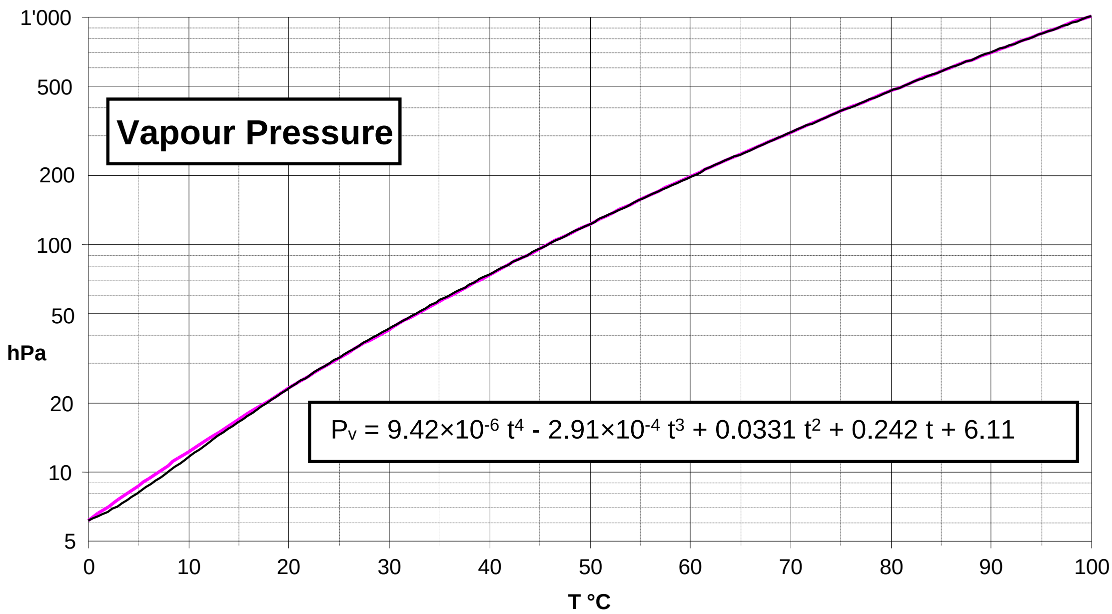
$$h = \frac{Pa - P_V}{\rho \cdot g} \quad \textbf{Eq. 2-4}$$

Thus when the temperature rises it will decrease the water column height and when it reaches the boiling point, P_V=Pa, the height of the column is zero.

This h gives also the maximum height to which a column of water can be raised or pumped. It will be about 10 m at 25°C at sea level, but will decrease to about 7.3 m for the same temperature at 2500 m above sea level. For higher temperatures it will decrease quickly, therefore pumping hot water has to be carefully done.

Fig 2- 3

Vapour pressure in hPa according to temperature



2.5. *Relative Water pressure **

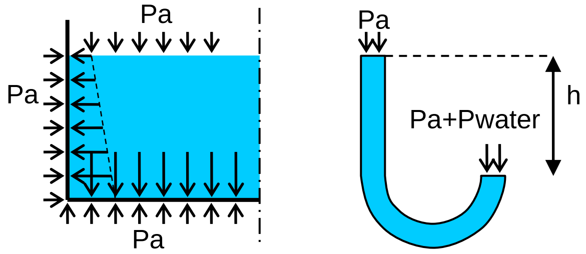
The total pressure in the water will be composed of the atmospheric pressure exerted at the surface of the water plus the pressure due to the water column.

P_{tot}: Total pressure [Pa]
P_a: Atmospheric pressure [Pa]
ρ: water density [kg/m^3]
g: is the earth's gravity [m/s^2]
h: depth of water [m]

$$P_{tot} = P_a + P_{water} = P_a + \rho \cdot g \cdot h$$ **Eq. 2-5**

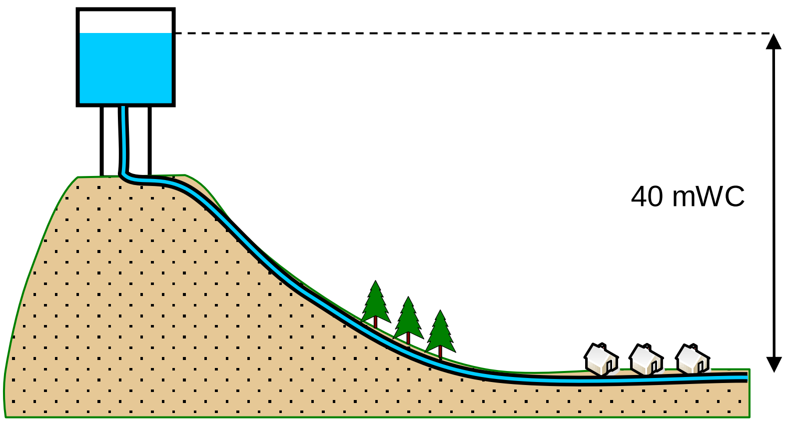
But if we look at the bottom or at the side of a water tank, the atmospheric pressure will act perpendicular to the surface, counterbalancing the effect of the pressure of the surface of the water. Thus the pressure on the side and the bottom of a tank will mainly be affected by the depth (h) of water.
Similarly, when we look at the pressure at the top of a pipe and at the bottom, both are subject to almost the same atmospheric pressure, therefore the available pressure will be the pressure due to the difference of water height.

$$\begin{aligned} P_{bottom} &= P_{tot} - P_a = P_a + \rho \cdot g \cdot h - P_a \\ P_{bottom} &= \rho \cdot g \cdot h \\ P_{bottom} &\approx 10'000 \cdot h \end{aligned}$$ **Eq. 2-6**

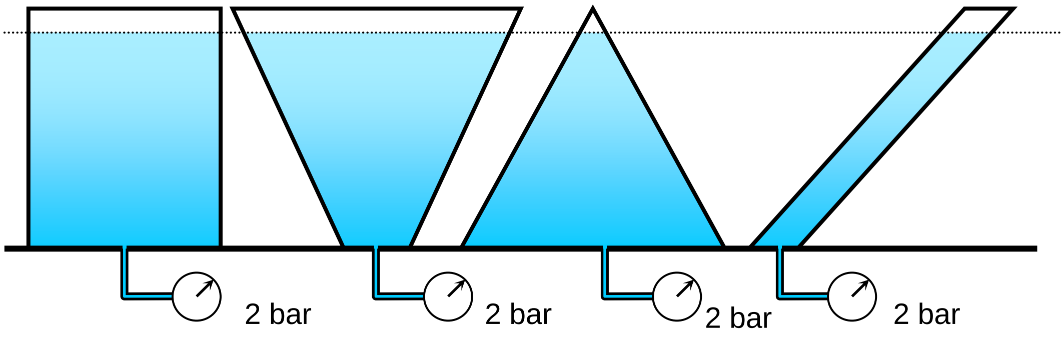


This is why in hydraulics, the pressure (P) refers to the water pressure and not to the total pressure. This is also why the pressure can under certain conditions be lower than 0 Pa.
For practical reasons, the units used are the bar or the meter of water column:
=> 1 bar ≈ 10 mWC

In a water supply system, pressure can be efficiently created by elevated tanks placed at a high position. In the following figures the static pressure provided by the elevated tank is 40 mWC or 4 bar



For house delivery, it is usual to have a pressure ranging from 3 to 5 bars.



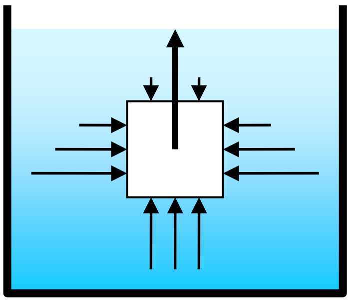
The pressure at the bottom of a reservoir is independent of its shape; for the same liquid level (height), the pressure at the bottom will be the same. This is important because in complex piping systems, it will always be possible to know the pressure at the bottom if the water height is known.

2.6. Archimedes' principle or buoyancy

Buoyancy is an upward acting force, caused by fluid pressure, which opposes an object's weight. In hydrostatics, the buoyant force is equal to the weight of fluid displaced by the body.

F: Buoyancy force in Newton [N]
ρ: density of the fluid [Kg/m^3]
V: volume of the body [m^3]
g: is the earth gravity

$$\boxed{\mathrm{F} = \rho \cdot \mathrm{V} \cdot \mathrm{g}} \quad \textbf{Eq. 2-7}$$



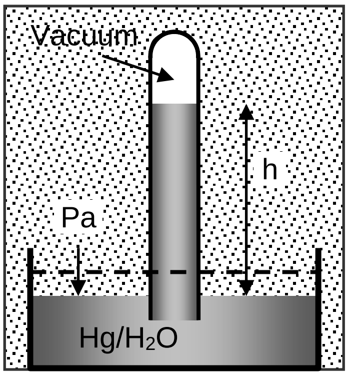
This force is always upward (i.e. opposing gravity) as the lateral forces cancel each other out.
If the object is either less dense than the liquid or is shaped appropriately (as in a boat), the force can keep the object afloat.

Basic exercises

1. What is roughly the pressure an 80kg man exerts on the ground?
2. What is the water pressure at the bottom of a pool 15 m deep at 5°C and at 25°C?

Intermediary exercises

3. What is the average atmospheric pressure at your place and at 4'000 masl?
4. At 4'000 masl, what is the elevation of a column (h) of mercury? and a column of water according to Eq 2-2?



5. What is the maximum suction height for water at 30°C at 1'500 masl?
6. What is the maximum suction height for water at 15°C at 3'000 masl?
7. What is the buoyant force applied to a body of 2 m^3?

Advanced exercises

8. A water pipe has its top at 500 masl and its bottom at sea level. What is the variation of atmospheric pressure between the top and the bottom? What is the water pressure at the bottom? What is the ratio between these two pressures?
9. What is the force applied by the liquid to the side of a water tank that is 2 m high and 10 m wide?
10. What is the buoyant force exerted on an 80 kg man by the atmosphere (Hint: we suppose that the density of a man is approximately the same as the one of water)?
11. An air balloon of 1m^3 at atmospheric pressure is brought at 10 m below water surface, what will be the buoyant force? And at 20 m below water surface?

Chapter 3. Hydrodynamics

What you will learn
You will understand the principles governing water dynamics within water supply systems.

Why
- An understanding of the physical laws that influence water supply systems is essential prerequisite in order to design efficient and long-lasting water supply systems.
- You will therefore receive basic knowledge on water flow rate, law of conservation of energy

Duration of chapter 3
2 - 3 hours

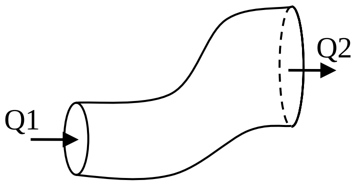
This chapter will study the general notions of hydrodynamics, derived from the law of conservation of energy. Reynolds number is also explained; although it looks a bit abstract, it is a very important concept governing fluid mechanics.
Practical situations will be treated in the two next chapters for piped (closed) and open flows.

*3.1. Flow **

According to the law of conservation of matter, for **permanent** conditions (i.e. not transient conditions such as water hammer), the mass flow rate is constant along a stream.

q: flow [kg/s]
ρ: density [kg/m^3]
A: cross section area [m2]
v: mean velocity of the fluid at a given i section

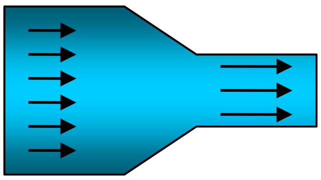
$$\boxed{q = \rho_i \cdot A_i \cdot v_i = cte} \quad \textbf{Eq. 3-1}$$



When the fluid is considered as incompressible, this equation can be simplified and a *volumetric* flow rate (m^3/s) can be used. This is the case for water and is also valid for air under low pressure, such as in building ventilation systems.
Across a given cross-section, velocity usually varies. It may vary from zero at a peripheral contact with a "wall" to a maximum at the centre. The velocity that will be used in these chapters is taken as the mean velocity across a given section. In the majority of cases encountered in hydraulics, velocity is fairly homogenous, this point will be further developed with the Reynolds concept.

Q: flow [m^3/s]
A_i: cross section area [m2]
v_i: mean velocity of the fluid at a given section, i.

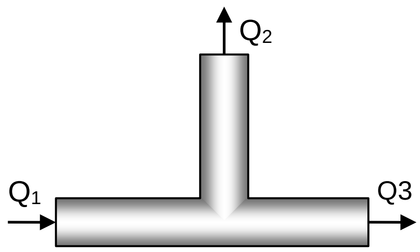
$$\boxed{Q = A_i \cdot v_i = cte} \quad \textbf{Eq. 3-2}$$



As shown above, for a given flow, the smaller the section, the greater the velocity passing through it.

If there are junctions, the law of conservation of matter stipulates that the total flow going in must equal the total flow going out.
In this example the flow (Q1) entering the pipe must equal the sum of the flow leaving (Q2+Q3)

$$\boxed{\sum Q_{in} = \sum Q_{out}}$$ **Eq. 3-3**



$Q_1 = Q_2 + Q_3$

For transient conditions, like water hammer, the water is still entering the pipe but no water is exiting. In this case, water will accumulate for a certain period inside the pipe and the previous equations are not valid during this period. This case will be further studied later on.

As the international system of units standard for flow (m^3/s) is quite large, more practical units are often encountered such as litres/second, litres/minute, m^3/day, etc…

3.2. Law of Conservation of Energy (Bernoulli's Principle) *

In hydraulics, the energy of a flow at a given point is composed of three main elements:
- the energy due to the pressure at this point or elastic energy,
- the potential energy according to the difference of altitude in the system,
- the kinetic energy due to the velocity of the fluid.

In the following expression, all of these elements are expressed in water column height [mWC] at a given point i :

E_i: energy
P_i: relative pressure
H_i: elevation
v_i: velocity of the fluid

$$\boxed{E_i = \frac{P_i}{\rho \cdot g} + H_i + \frac{v_i^{\,2}}{2g}}$$ **Eq. 3-4**

This equation is only valid for permanent, incompressible flow with negligible changes of internal energy (temperature) and without vortex.
NB: H is calculated from an arbitrary reference point (which is usually either the sea level or the lowest point occurring in calculations) but should stay coherent all along calculations.
According to the law of conservation of energy, the energy at point A equals the energy at point B minus any losses between these two points. The energy can change from one form to another, for instance from mainly potential energy to pressure or to kinetic, but the total will always respect the law of conservation of energy.

E_A: total energy at position A
E_B: total energy at position B
$H_{A\text{-}B}$: energy losses between A and B

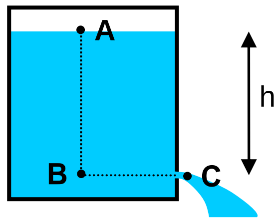
$$\boxed{E_B = E_A - H_{A-B}}$$ **Eq. 3-5**

The evaluation of any losses is the most complicated problem, because it cannot be deduced from mathematical equations. Therefore it has to be calculated through experiments, from which empirical laws are developed. The most commonly encountered formulas are Chezy-Manning, Darcy-Weisbach and Hazen-Williams; their use will be reviewed in chapter 4.5. For the following examples, we will neglect losses in order to observe particular applications of Bernoulli's Principle.

For a large tank with a small orifice at the bottom, we can consider the energy at point A as being only potential, since the velocity is negligible and the water pressure is zero (E_A).
Points between A and B gradually lose potential energy and increase pressure. At point B, all potential energy is lost, the velocity is still negligible but there is significant pressure (E_B).
Points between B and C gradually increase in speed and lose pressure until the orifice (point C), where there is no more water pressure, no more potential energy and only velocity. Neglecting the losses, we can say that all the initial potential energy was ultimately transformed to kinetic energy (E_c).
Therefore, from A to B to C the energy type changes but the total energy is conserved.

$$E_A = h$$
$$E_B = \frac{P_B}{\rho \cdot g} = h$$
$$E_C = \frac{v_C^2}{2 \cdot g} = h$$
$$v_C = \sqrt{2 \cdot g \cdot h}$$

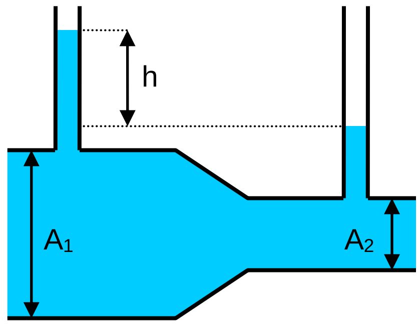
Torricelli's theorem



Venturi effect

The Venturi effect can be used to measure the velocity of a fluid by comparing the pressure difference between two known sections of pipes.
Thanks to Eq. 2-5, Eq. 3-2 & Eq. 3-4, neglecting the losses the following speed can be calculated according to the difference of height (h).

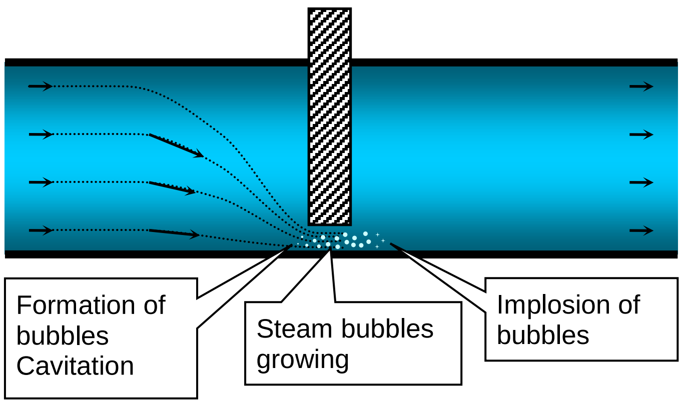
$$v_1 = \sqrt{2 \cdot g \cdot h / \left((A_1/A_2)^2 - 1 \right)} \quad \textbf{Eq. 3-6}$$



This is a very cheap and efficient way to measure flow rate. Transparent hoses or glass pipes are necessary to measure the difference of level and need to be regularly cleaned.
When the section A_2 is small enough, air can be sucked into the pipe by the passing water, this principle is used in several practical applications, such as in vacuum pumps or chlorine injectors. When air is sucked in the water, air bubbles will be taken in to the flow and remain there unless absorbed. In this case, we have a compressible fluid with a mixture of liquid and gaseous phases and the equations seen previously are no longer valid.

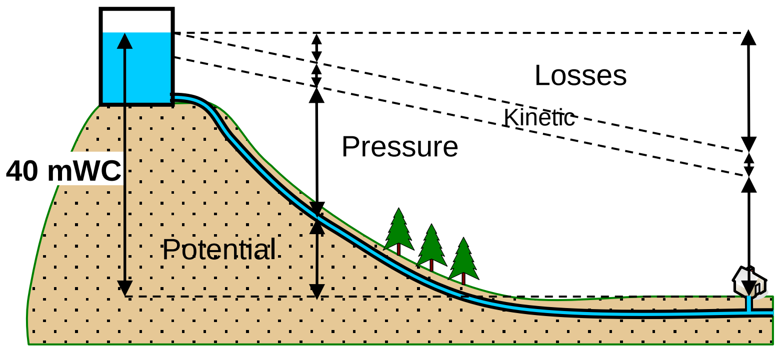
Cavitation while passing a gate valve or an obstruction

When a section in a pipe is very small, such as at a gate valve that is almost closed, the pressure can decrease until it reaches vapour pressure (as the velocity increases). At this stage, the water will start to cavitate and a typical noise (like small stones hitting the pipe) can be heard.



Cavitation produces small vapour bubbles (not air bubbles) that might grow a bit if the velocity further increases. After the restriction, when the velocity decreases again, the pressure rises and the bubbles implode suddenly, progressively eroding the pipes downstream of the restriction. It is these implosions that make the typical cavitation noise. Similar phenomena can occur in open flow in dams' spillways, and the effect can easily destroy concrete structures.
When cavitation occurs, the equations seen previously are not applicable as we have two phases present: compressible gas and water; the pressure will be kept at its vapour value. In most simulation software (like Epanet) this situation cannot be taken into consideration and a warning message is given but the equations are still solved with negative values for the pressure.

Representation of energy (or piezo) line



As all components of energy are expressed in meters of water column, they can easily be represented as elevations on a diagram. In this way, it is easy to see and understand the transformation of energy from potential to kinetic and pressure and to see the losses.
The example represented in the above figure shows a simple piped scheme running from an elevated tank to a house. At the top of the tank all of the energy is potential. When entering the pipe, it takes on speed and so part of the available energy is kinetic. Along the pipe, the kinetic energy will be constant if the pipe diameter is constant, and the pressure will rise as potential energy decreases, and a part of the available energy will be lost in friction losses. At the delivery point, all of the initial potential energy has been transformed into lost, kinetic and pressure energies.

3.3. Reynolds number

In fluid mechanics, the Reynolds number (Re) is a dimensionless number that indicates the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions.
It is used to characterize different flow regimes, laminar or turbulent flow. Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion. While turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.

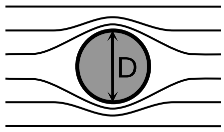
Reynolds number depends only on three values:
- the viscosity, fixed for a given fluid at a certain temperature
- the velocity
- the specific dimension (the internal diameter for a pipe, the thickness for a wing, etc).

Re: Reynolds number
D: specific dimension [m]
v: velocity of the fluid [m/s]
ν: kinematic viscosity

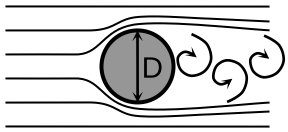
$$\boxed{\mathrm{Re} = \frac{\mathrm{D} \cdot \mathrm{v}}{\nu}}$$

Eq. 3-7

Laminar flow around a body: in this situation each flow line returns to its initial position after the obstacle. There is no mixing in the fluid and velocities are changed smoothly to pass the obstacle and find their original values after it.



Turbulent flow around a body: in this situation the flow is disturbed over a long distance, the flow lines are mixed in vortices formed after the obstacle. The velocities are changing quickly when approaching the obstacle.



***Transient conditions**: Flow can only be laminar or turbulent; there is no intermediate flow regime between these conditions. Flow is considered to jump from one state to the other. The exact Re values of this jump might vary with some parameters like roughness, or speed of change. It might also not be the same value when passing from laminar to turbulent and from turbulent to laminar. Therefore the transition Re is given over a wide range (23'000 to 40'000 for pipes).*

Basic exercises

1. What is the flow in a pipe of 150mm of diameter with a 1m/s speed?
2. What is the speed in the same pipe after a reduction in diameter to 100mm and to 75mm?
3. What is the flow in the same pipe after a Tee with a 25mm pipe with a 1 m/s speed?
4. What is the speed of water going out of the base of a tank with 2.5m, 5m and 10m height?
5. What are the speed and the flow in a pipe of 150mm of diameter showing a difference of 20cm height in a Venturi section of 100mm diameter?

Intermediary exercises

6. What are the Reynolds numbers for the flows in the exercises 1,2 & 5?
7. For a pipe of 150mm of diameter, with 2 bar pressure and a a velocity of 1 m/s, what should be the diameter reduction to cause cavitation at 20°C (neglect head losses)?

Advanced exercises

8. With equations Eq. 2-5, Eq. 3-2 & Eq. 3-4, demonstrate Eq. 3-6.
9. For a pipe of 150mm of diameter, with 1 bar pressure and a velocity of 1 m/s, what should be the diameter reduction to cause air suction?
10. A tank 2m high and 1m diameter has a 75mm valve at the bottom. When the tank is full and the valve is quickly opened, how long does it take to empty the tank (the losses and contraction factor are neglected)? (Difficult exercise to be solved with integral calculations)

Chapter 4. Flow in pipes under pressure

What you will learn
Know the different specifications for pipes
Understand the pressure rating
Know the types of flow in a pipe
Understand the concept of friction losses in pipes under pressure

Why
Be able to calculate the punctual and linear losses in pipes under pressure

Duration of chapter 4
3 to 4 hours

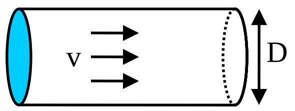
*4.1. Pipe diameters and PN **

The section of a cylinder is defined as follow:

A: Cross section area [m^2]
D: Diameter [m]
Q: volumetric flow [m^3/s]
v: velocity [m/s]

$$A = \pi \cdot \frac{D^2}{4}$$
$$Q = \frac{v \cdot \pi \cdot D^2}{4}$$

Eq. 4-1



NB: D is usually given in millimetres but "inches" is still quite often used (1" ≈ 25mm). Therefore, when using this formula, it should be checked that D is converted in meters.

However dimensions of a pipe are more complicate and depend on material, pressure rating and temperature, etc. The base notions for all of them are:

Outside diameter (OD): it is the diameter of the whole pipe including coating expressed in millimetres. As pipes are usually mate by the outside, it is the dimension interesting the plumber. It is defined by standard for all types of pipes (ISO 161-1 for metric).

Internal diameter (ID): it is the diameter of the hollow part of the pipe where water can flow. Therefore it is the diameter to be used for calculation, meaning the diameter interesting the designer.

Generally the OD is defined in standards; the ID should therefore be deducted from the OD and the thickness, which depends on the material and usually also the pressure rating. There is two ways to define pipes dimensions, they are developed below according to international standards, but many countries have different or "free" standards therefore, always ask all specification to the pipe supplier and cross-check them carefully.

Plastic pipes

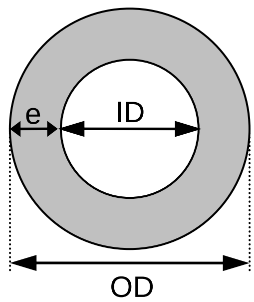
The first way to define pipe dimensions uses the OD as reference diameter, it is generally used for plastic pipes (like PVC and PE) which have a thickness varying a lot with the rated pressure. The thickness (e) is then defined with the Standard Dimension Ratio (SDR) or the pipe series (S) accordingly:

OD: outside diameter [mm]
ID: internal diameter [mm]
e: wall thickness [mm]
SDR : standard dimension ratio
S: pipe series

$$\boxed{ID = OD - 2 \cdot e} \quad \textbf{Eq. 4-2}$$

$$\boxed{SDR = \frac{OD}{e} = 2 \cdot S + 1} \quad \textbf{Eq. 4-3}$$

$$\boxed{S = \frac{OD - e}{2e} = \frac{SDR - 1}{2}} \quad \textbf{Eq. 4-4}$$



Practically each supplier gives a table with the thickness of the pipe for each OD according to the available SDR. The thickness (e) is usually rounded at the higher value according to the tolerances.

The nominal pressure (PN) indicates the maximum working pressure for a pipe. For plastic pipes, it depends on the material, a safety factor and also the temperature of the fluid if it exceeds 25°C.

PN: Nominal Pressure [Bar]
MRS: minimum required strength [MPa]
S: pipe series
C: Service ratio

$$\boxed{PN = 10 \cdot \frac{MRS}{S \cdot C}} \quad \textbf{Eq. 4-5}$$

For **PVC** pipes (annexe C) according to ISO 4422-2 : the MRS should be of a minimum of 25 MPa for a design life of 50 years.

For PVC pipes of OD 90 or less, C is taken as 2.5

SDR	41	34.4	33	26	21	17	13.6	9
S	20	16.7	16	12.5	10	8	6.3	4
PN	5	6	6.3	8	10	12.5	16	25

For PVC pipes of OD 110 and larger C is taken as 2

SDR	41	33	26	21	17	13.6	11
S	20	16	12.5	10	8	6.3	5
PN	6.3	8	10	12.5	16	20	25

A supplementary derating factor (f_t) shall be applied if the fluid temperature is between 25°C and 45°C: PN*=PN·ft

T [°C]	25	30	35	40	45
$\mathbf{f_t}$	1.00	0.88	0.78	0.70	0.63

For **Polyethylene** (annexe B) pipes C is taken as 1.25 for drinking water, according to DIN-8074,

For PE80, the MRS is 8 MPa

SDR	51	41	33	26	22	21	17.6	17	13.6	11	9	7.4	6	5
S	25	20	16	12.5	10.5	10	8.3	8	6.3	5	4	3.2	2.5	2
PN	2.5	3.2	4	5	6	6.3	7.5	8	10	12.5	16	20	25	32

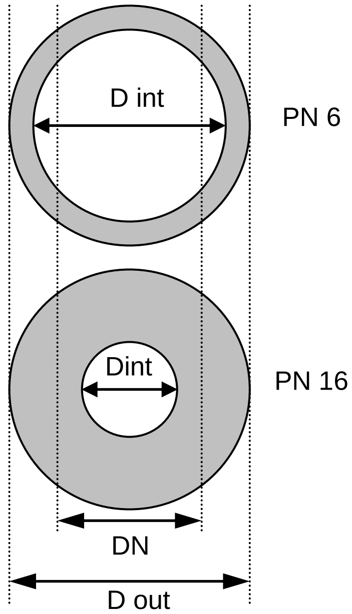
For PE100, the MRS is 10 MPa

SDR	51	41	33	26	22	21	17.6	17	13.6	11	9	7.4	6	5
S	25	20	16	12.5	10.5	10	8.3	8	6.3	5	4	3.2	2.5	2
PN	3.2	4	5	6.3	7.5	8	9.6	10	12.5	16	20	25	32	40

NB: Sometimes the OD is also called Nominal Outside Diameter, this refers to the production tolerance and means that it is the minimum acceptable diameter. The variation might be up to 0.3% more with a minimum value of 0.1mm and a maximum value of 2mm (ISO 11922-1 Grade C).

The derating factor for pressure on temperature higher than 25°C might vary with the supplier.

Metallic pipes,



The second way to define pipe dimensions is for metallic pipes but was also used for PVC pipes. The thickness are generally thin and homogenous (it varies just a bit between the different pressure rating), thus an internal diameter is used as reference although it might not be the actual internal diameter: it is defined as the **Nominal diameter (DN):** "It is a number that approximates bore diameter measured in millimetres, and is only loosely related to manufacturing dimensions."
For a given DN, OD will be the same but the ID will vary with the PN. Thus the DN represents a "compromise", might be a minimum or an average depending on the material used. Therefore, the exact internal diameter should be crosschecked with the supplier. If the pipe is coated, it should be usually deduced form the internal diameter.

For instance for a DN100 the OD is about 114 mm :

- For a GI pipe there is 3 series, light up to 6 bar, medium up to 10 bar and heavy up to 16 bar with an ID of respectively 107, 105 and 103 mm
- For a PVC pipe for the same pressure the ID are 108, 104 and 99 mm

For **cast iron pipes**, as they are very hard, the thickness does not vary with the PN, thus the nominal and the internal diameter are the same. The only variation between different PN pipes are the flanges, for instance a DN200 PN 10 has 8 bolts as the PN 16 has 12.

4.2. Hydraulic diameter

Most of the tables and equations give values according to diameters for round pipe. In the cases where the pipe is not round, for instance square or squeezed, an equivalent diameter has to be used and is call the hydraulic diameter (D_h). Once the D_h has been found, it can be used as a usual diameter in all formulas and tables.

D_h is defined as four times the ratio of the area by the perimeter:

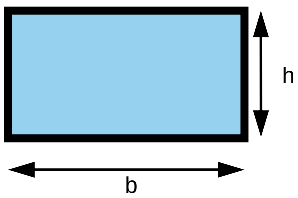
D_h: Hydraulic diameter
A: section [m^2]
P: wetted perimeter [m]

$$D_h = 4\frac{A}{P}$$ **Eq. 4-6**

For a rectangular section pipe :

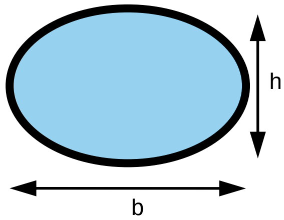
h: Height of the pipe
b: width of the pipe [m]

$$A = b \cdot h \qquad P = 2 \cdot b + 2 \cdot h$$ **Eq. 4-7**



For an elliptic section pipe :

$$A = \frac{\pi}{4} b \cdot h \qquad P = \frac{\pi}{2}(b + h)$$ **Eq. 4-8**



The hydraulic diameter for closed pipes is not the same as the hydraulic radius for open flows: D_h is not equal to 2 hydraulic radiuses! Cf chapter 5

4.3. Type of flow

As it is explained for the Reynolds number, the type and profile of the flow will depend on the Reynolds number. In a pipe, the diameter to use is the internal or hydraulic diameter and the speed is the average speed for a given section.

Re: Reynolds number
D: specific dimension [m]
v: velocity of fluid [m/s]
ν: kinematic viscosity
≈10^{-6} [m^2/s] at 20°c

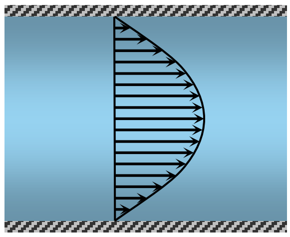
$$Re = \frac{D \cdot v}{\nu}$$

$$Re = \frac{4 \cdot Q}{\pi \cdot D \cdot \nu}$$

Eq. 4-9

Laminar flow (Re < 2'000)

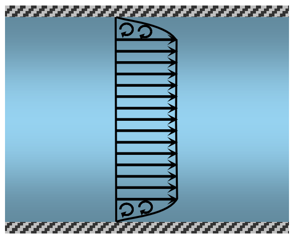
In this case, the water behaves like blades flowing on top of each other's. Completely stopped at the side and with a maximum velocity in the middle; it has a parabolic profile.
The average velocity = 5/6 Vmax
As the velocity is limited at the sides, particles can settled and bio film can developed easily. Therefore, the pipes might get clogged or contaminated quickly.



In a water system, the flow should not be laminar.

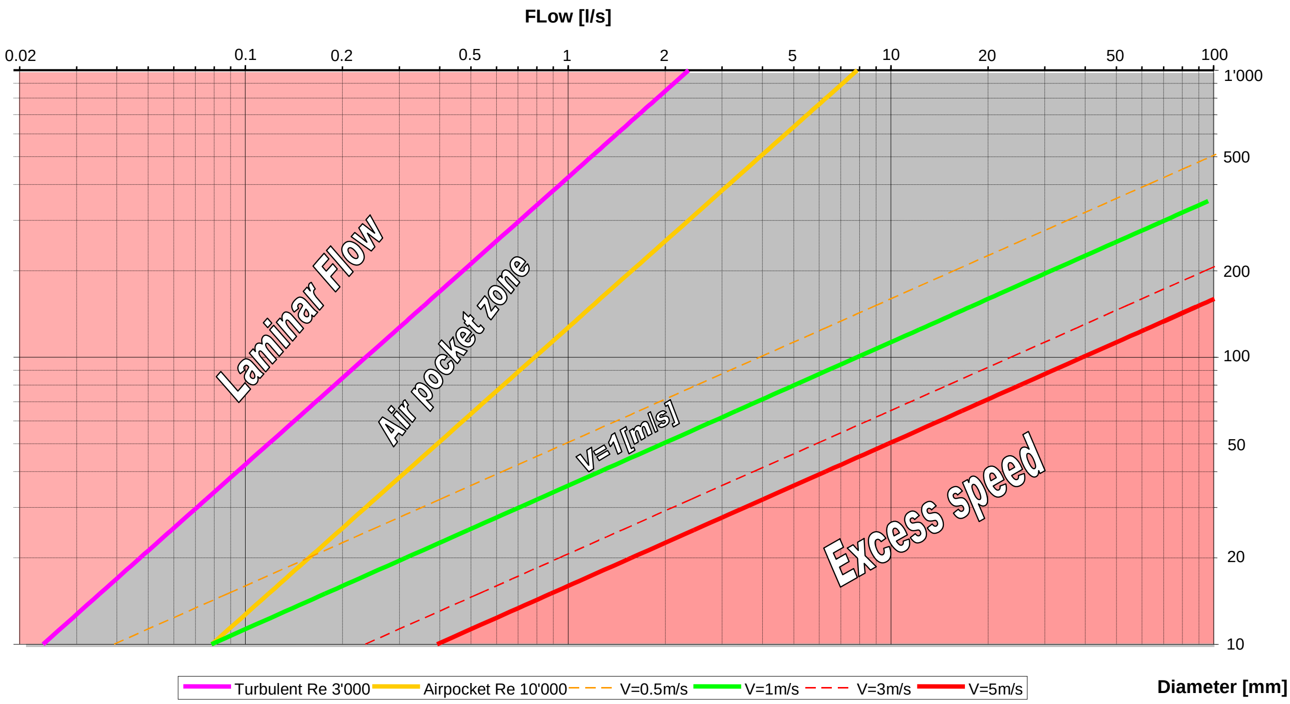
Turbulent flow (Re> 3'500)

In this case, on the sides, small vortexes are acting as ball bearing facilitating the flow, and in most of the sections, the velocity is uniform; it has an exponential profile.
The average velocity ≈ Vmax
Thanks to the small vortexes, all settled particles are easily removed and the bio-film cannot develop itself.



In a water system, the flow should be turbulent.

It was also experienced that with a higher Reynolds (~10'000), the flow will ensure that air bubbles are taken away, thus the installation of air valve are not needed during operation.



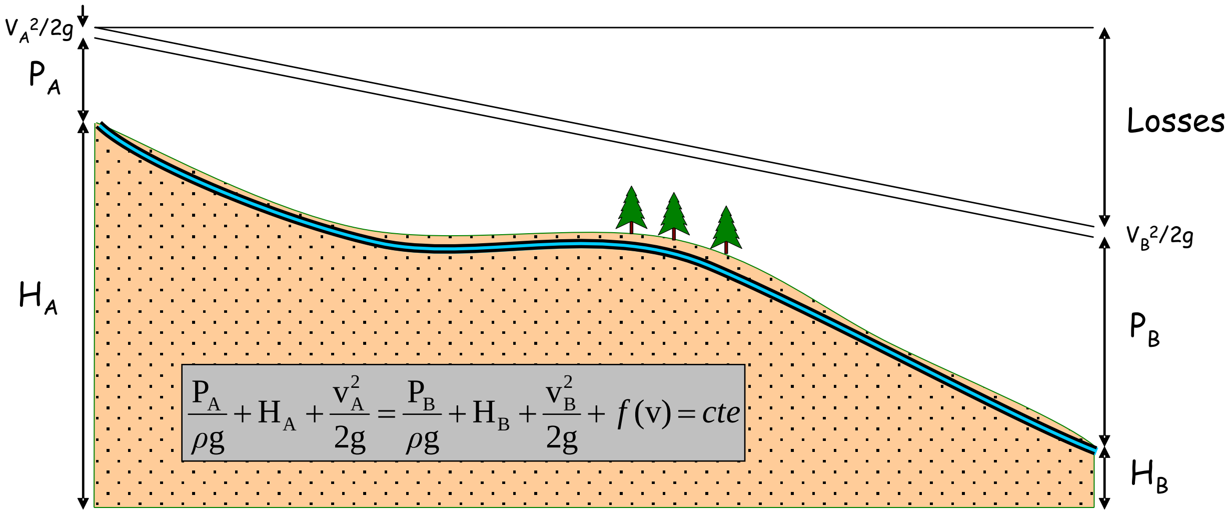
In the previous chart, the limit between laminar and turbulent flow is represented with the purple line, the area delimited with the yellow line represents the condition where air pocket might stay in pipes.
The maximum velocity is usually taken between 3 to 5 m/s (red line) and depends on the capacity of the pipe to stand erosion. The velocity has also an important impact on water hammer and could also be limited by this factor as explained in chapter 7.

The green line represents the flow at 1 m/s, usually used as a base for design. However we can see in the above chart, that for small diameter, higher velocity should be preferred as for larger diameter, lower velocity can be used without problems.

As the viscosity varies quite a lot with temperature, the Reynolds will also be very sensitive, and will for instance increase of 44% between 0 and 20°C.

We can also see in the chart that for small diameter, using a velocity higher than 1 m/s will avoid to reach air pocket zone. As for larger diameter, lower velocity can be used without risks.

4.4. Energy or piezo line for pipes under pressure *

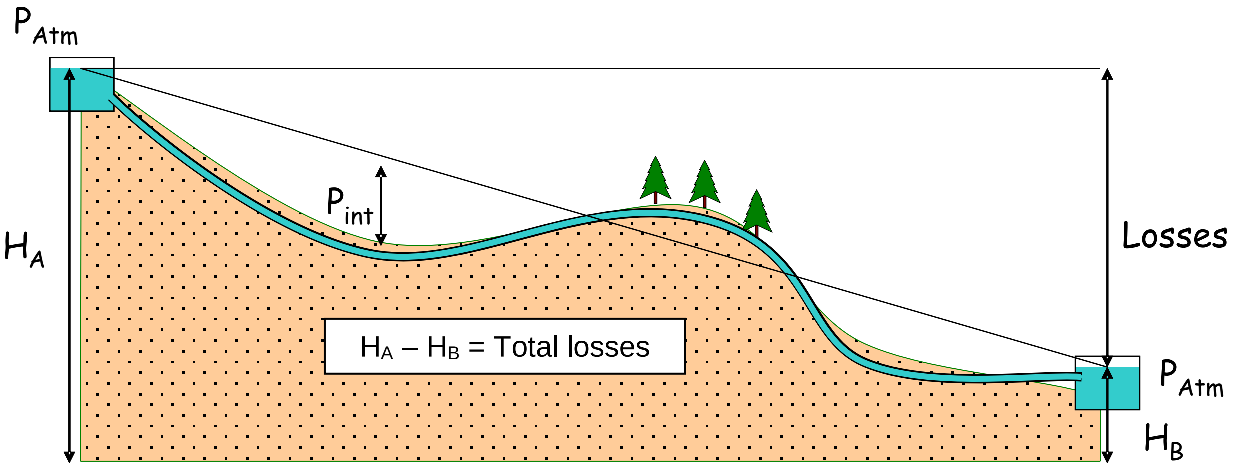


$$\frac{P_A}{\rho g} + H_A + \frac{v_A^2}{2g} = \frac{P_B}{\rho g} + H_B + \frac{v_B^2}{2g} + f(v) = cte$$

The above figure represents the energy line for a pipe section with initial and residual pressure. The initial energy is formed by its pressure (P/ρg), potential (H) and kinetic ($v^2/2g$) parts. The kinetic energy can generally be neglected, for instance with a velocity of 1 m/s it represents a height of 5cm. At the end of the section, a part of this energy is dissipated into friction losses: f(v). If at any point a vertical pipe is connected, the water level will reach the piezo line level.

If the water is flowing from one tank or intake chamber A to a lower tank B, the pressure velocity at these points is negligible thus, the potential energy (difference of height) is **entirely** dissipated on losses. If the whole pipe is full the energy line can be found with the equations of this chapter, if all or part of it is partially full (open flow) the energy line will be almost at the same level as the pipe. This case is further developed in the next chapter.

We can see also that if the energy line is straight, it might pass under the pipe altitude, in this case the pressure might be negative (the water is no compressed but dilated) if this "negative" relative water pressure exceed the vapour pressure cavitation will occur. In this situation, the fluid will have two phases flow (liquid and gas) and it is not possible to calculate any more its behaviour. However, its pressure can never go below vapour pressure level.



4.5. Friction losses *

Friction losses cannot be deducted from the base equations of hydraulics. The only way to predict them is by making tests in laboratory and try to generalise the results. Therefore no friction losses equation are exact, they are just more or less precise according to the situation.
The famous "couples" that have tried to define those losses are Chezy-Manning, Hazen-Williams, and Darcy-Weisbach. For pipes under pressure, the system developed by Darcy-Weisbach is the most practical and intuitive and will be used in this course. They started from the observation that losses where generally proportional to the square velocity: $H_{Losses} = f(v^2)$ and can be separated between linear losses, happening along pipes, and punctual losses, happening in fittings or other punctual phenomenon in the flow. To have a dimensionless coefficient the square velocity is divided by "2g", it gives the following equation for a section with a given velocity:

h_{LP}: hydraulic losses [mWC]
k_{LP}: friction coefficient [-]
v: velocity [m/s]

$$h_{LP} = k_{LP} \cdot \frac{v^2}{2 \cdot g} \quad \textbf{Eq. 4-10}$$

For a pipeline with a given flow going form point A to B, the losses follow the equation:

Q : flow [m³/s]
D: pipe's diameter [m]

$$H_A - H_B = \sum h_{LP} \cong \frac{Q^2}{12.1} \cdot \sum \frac{k_{LP}}{D^4} \quad \textbf{Eq. 4-11}$$

It shows that the losses are proportional to the power four of the diameter and only to the square for the flow, thus a small change on diameter has a great impact on the flow.

4.6. Punctual friction losses

Those losses take place when an "accident" occurs to the flow, disrupting the velocity profile. It comprises changes of direction (elbow, tee), changes of sections (extension, reduction, inlet, discharge) and obstacles in the flow (valves, filters, orifice).
The value of the punctual friction losses coefficient k_p is constant for a given fitting. Their theoretical values can vary quite a lot from one table to an other. The best is to ask the supplier to give the actual value for their fitting, especially for valves.
It is not advisable to take a ratio of the linear losses as part of the punctual losses as their share might vary a lot according to the system configuration.

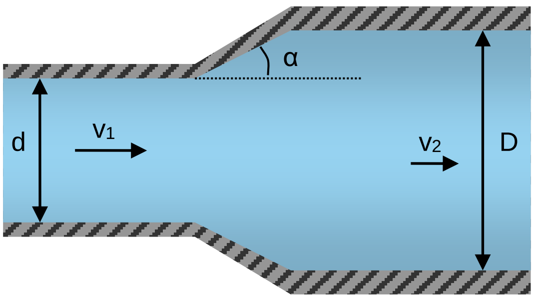
The following table summarised some of those losses coefficient.

Table 4- 1 punctual loss coefficient

Types	k_P			Comments
	Min	Average	Max	
Inlet	0.05	0.5	3	Bevel to reduce
Discharge	1	1.5	2	Place after good length of straight pipe
90° square elbow	1.15	1.2	1.3	Add fairing at the inner angle to reduce
90° round elbow	0.1	0.35	0.5	The curve should be ~ 2D
Tee flow in line		0.35		According to the % of flow
Tee line to branch	0.8	1	1.3	According to the % of flow
Ball valve	0.02	0.1	0.15	Decreases with the size (DN)
Gate valve	0.1	0.35	0.6	
Butterfly valve	0.06	0.5	2	
Globe valve	3	5	6.8	
Non-return valve	1.5	2.5	6	
Angle valve	2	4	6.6	
Foot valve with strainer		15		Might vary a lot
Filters	1	2.8	5	In clean condition
Water meters	7.5	10	15	Should be given by the manufacturer

The **theoretical** friction losses coefficients for enlargements, contractions and orifices are according to the following charts:

For an enlargement



$$h_{P\ Enlargment} = k_{p\text{-}e} \cdot \frac{v_1^2}{2 \cdot g}$$ **Eq. 4-12**

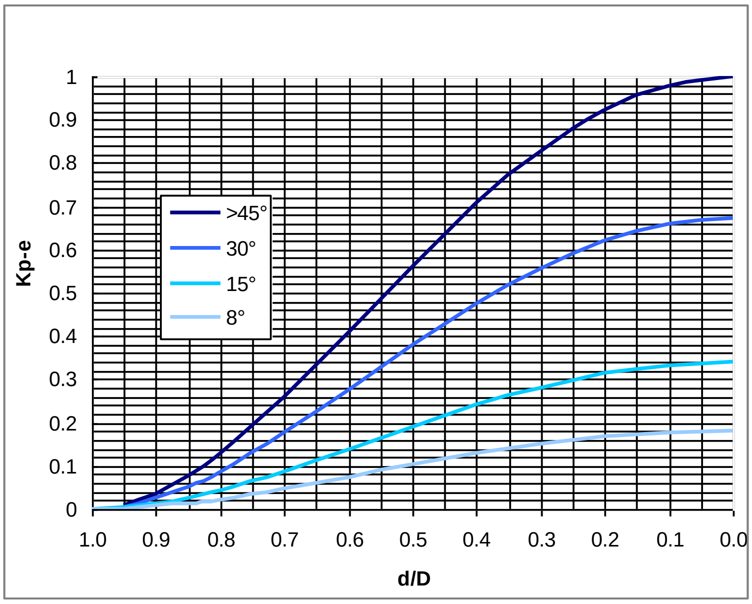
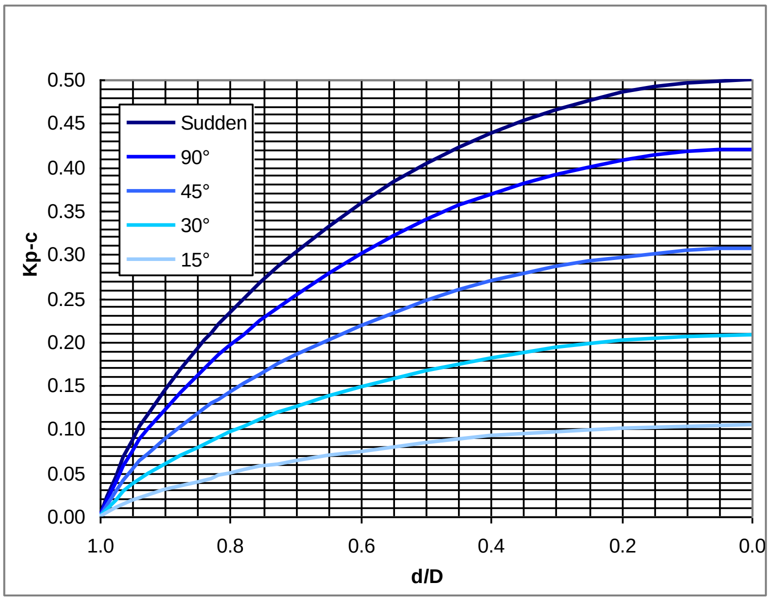
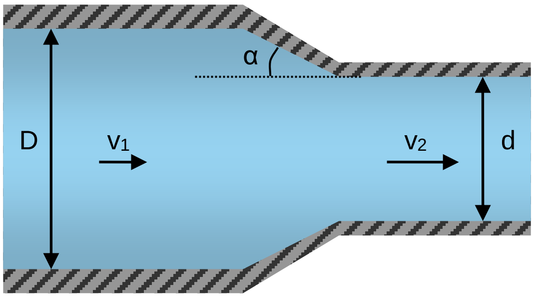


Fig 4- 1 Head losses due to enlargement



For a contraction :

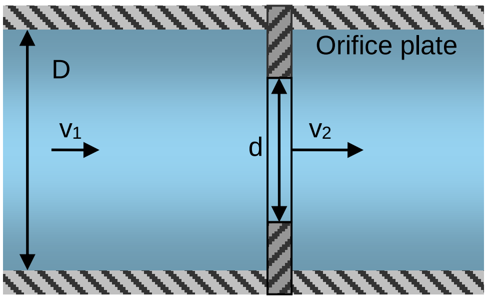


$$h_{P\ Contraction} = k_{p\text{-}c} \cdot \frac{v_2^2}{2 \cdot g} \quad \text{Eq. 4-13}$$

Fig 4- 2 Head losses due to contraction

NB: the Kp is to be applied at the velocity of the section of the smaller diameter

For an orifice :



$$h_{P\ Orifice} = k_{p\text{-}o} \cdot \frac{v_1^2}{2 \cdot g} \quad \text{Eq. 4-14}$$

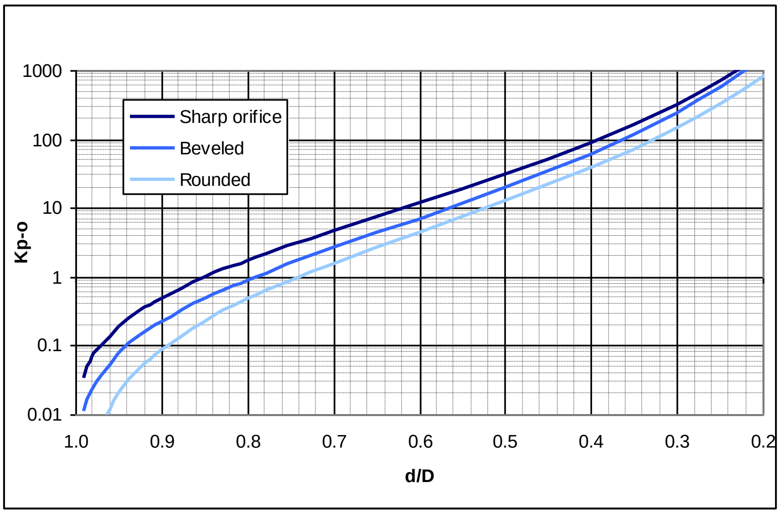


Fig 4- 3 Head losses due to orifices

In the case of contraction and orifice, there is depression due to the acceleration of the water. Thus cavitation limits the minimum d/D ratio around 0,3 to 0,2.

A concrete application where head losses are found due to orifices are polyethylen pipes. Indeed, when PE pipes are welded, they form a bead, which in turn will produce head losses, as they act like an orifice. Usually, head losses due to welding are included in the linear friciton loss coefficient.

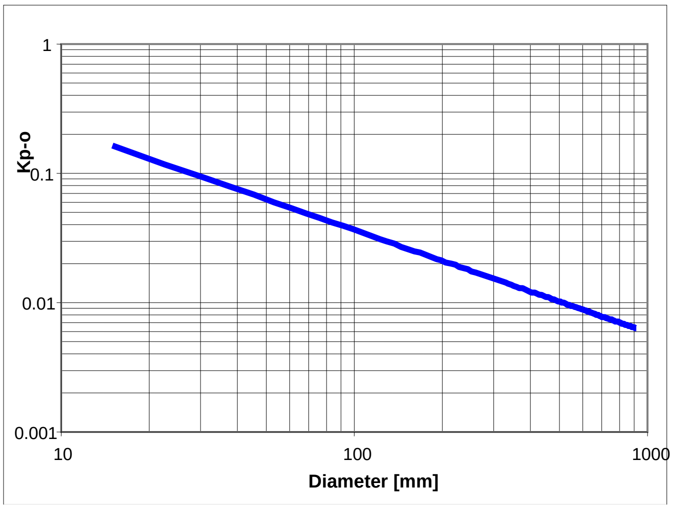


Fig 4-4 Punctual losses coefficient due to bead in PE pipe

In this chart, we can see that head losses are quite big for small diameters. On the other hand, this has only a small influence on big diameters. This is because the size of the bead does not vary a lot as the diameter increases. Therefore, for small diameter, the bead will fill up a bigger portion of the pipe. Welded PE conducts are therefore not recommended for pipes with diameter smaller than 50 mm, because the head losses would be too high.

4.7. Linear friction losses

The coefficient of linear friction losses (k_L) was found proportional to the length of the pipe and inversely proportional to the diameter, but also very influenced by the type of flow (link to the Reynolds and the roughness). The following equation defines the coefficient of linear friction losses with the length and its diameter and an additional coefficient taking into account the flow type: λ
The **usual roughness** for pipes is as follow:

Table 4- 2 Linear losses coefficient

Material of pipe	k in [mm]		
	Min	Aver.	Max
Steel new riveted	1		10
Steel new GI	0.1	0.12	0.15
Cast iron uncoated	0.15	0.22	0.30
Cast iron coated		0.12	
Steel moderate rusty	0.1	0.15	0.2
Steel slight incrustation	0.2	0.3	0.4
Steel heavy incrustation	0.4	1	4
Concrete factory made	0.1	0.2	0.5
Concrete made on site	0.3	1	3
Asbestos cement	0.03	0.07	0.1
PVC		0.005	
PE	0.001	0.007	0.015
Copper, lead, brass	0.001	0.0015	0.002

k_L: linear losses [-]
L: length of pipe [m]
D: diameter of pipe [m]
λ: coefficient of linear losses [-]

$$k_L = \lambda \cdot \frac{L}{D}$$
$$h_L = \frac{v^2}{2g} \cdot \lambda \cdot \frac{L}{D} \quad \text{Eq. 4-15}$$

Moody chart: This chart represents the linear losses coefficient according to the Reynolds number and the roughness of the pipe. Five different area are represented

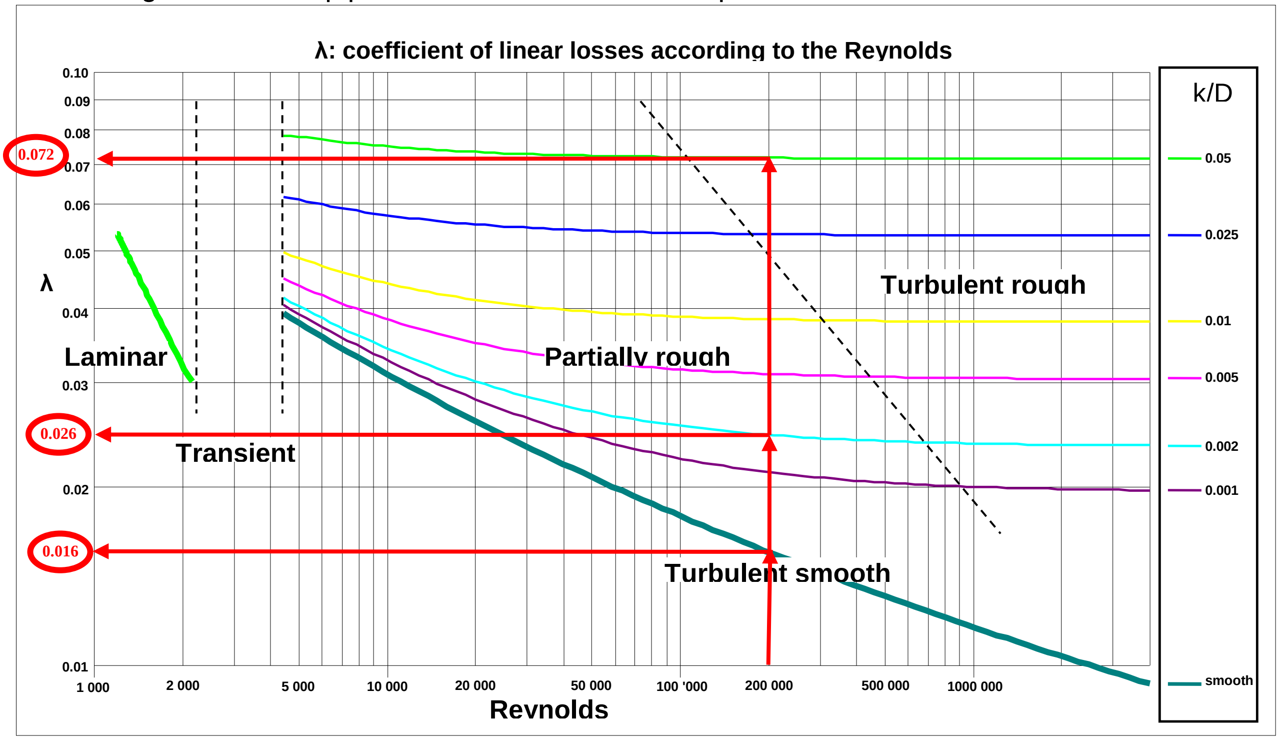


Fig 4- 5 Moody diagram

Example on the use of moody diagram:
In the three following cases, we have a Renold's number of 200 000 and a DN100
A: The pipe is smooth. From a Reynolds on x-axis of 200 000, a line is drawn vertically until it crosses with the grey-green line corresponding to smooth pipe. A line is then drawn horizontally and a λ value of 0.016 can be read on the y-axis.
B: The pipe has a roughness k of 0.2 mm. The k/D ration must be calculated: k/D=0.2/100=0.002. The vertical line is drawn until it intersects the blue line corresponding to k/D=0.002 and the value of λ can be read at 0.026.
C: The pipe has a k of 5 mm. k/D=0.05. The corresponding line is the bright green line and the λ value is 0.072

Laminar :
When the flow is laminar (low velocity) the roughness has no impact on the losses, they are due to the friction between the different blades sliding on each other, the coefficient is therefore quite high and inversely proportional to the Reynolds:

$$\lambda = \frac{64}{\mathrm{Re}} \quad \textbf{Eq. 4-16}$$

Turbulent rough:
When the flow is turbulent but the roughness of the pipe is important towards the diameter, the flow is mainly influenced by this effect and the coefficient depends only on the ratio D/k.
In this equation k and D should have the same unit !

$$\lambda = 0.0055 + \frac{0.15}{\sqrt[3]{\frac{D}{k}}} \quad \textbf{Eq. 4-17}$$

k : average pipe roughness [mm]
D: diameter of pipe [mm]

Turbulent smooth :
When the pipes are smooth as it is generally the case for plastic pipes, losses are only due to hydraulic frictions, therefore only depending on the Reynolds number.

$$\lambda = \frac{0.309}{\left(\lg\frac{\mathrm{Re}}{7}\right)^2} \quad \textbf{Eq. 4-18}$$

Global :
A complicated but handy equation includes all the above-mentioned situations, including partially rough conditions, it can be used easily in excel or other computer calculation software. The results for transient condition (laminar/turbulent) should be used with care.
In this equation k and D should have the same unit !

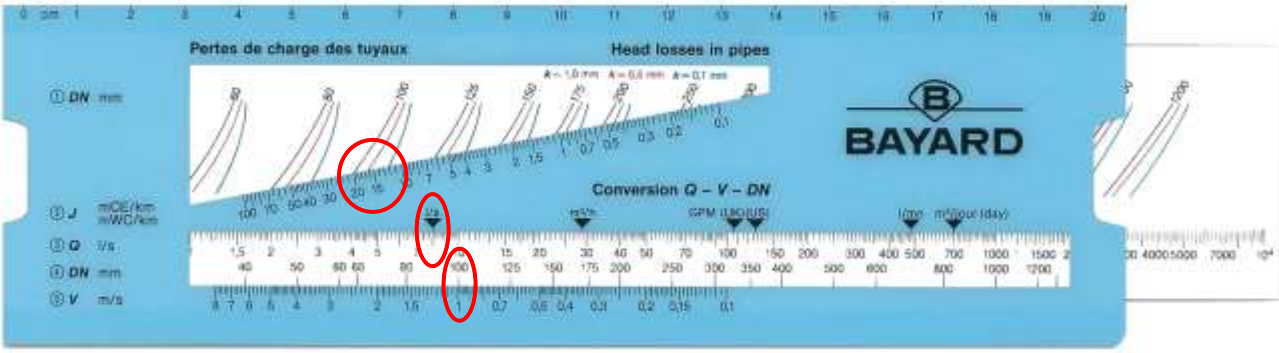
k : average pipe roughness [mm]
D: diameter of pipe [mm]

$$\lambda = 8\cdot\left(\left(\frac{8}{\mathrm{Re}}\right)^{12} + 1\Big/\left(\left(2.457\cdot Ln\left(1\Big/\left(\left(\frac{7}{\mathrm{Re}}\right)^{0.9}+0.27\cdot\frac{k}{D}\right)\right)\right)^{16}+\left(\frac{37530}{\mathrm{Re}}\right)^{16}\right)^{1.5}\right)^{1/12} \quad \textbf{Eq. 4-19}$$

4.8. Bush methods for friction losses *

There is some other ways to calculate it "by hands". Those methods were developed long time ago when computers were not available. Nowadays they should only be use to do quick calculation in the field or to cross check results. In this section, three methods are described to estimate linear friction losses. To account for punctual friction losses, roughly 5% (for long pipe system) to 10% (for short pipe system) must be added to linear friction losses.

The **Log Slide Rule** is a simple and quick way to find the losses or the needed diameter. The reading is done by moving the insert to the desired position, and then the other values can be directly read. It is quite simple and a bit more precise than the Log scale (see next page) quite useful for metallic pipe (limited number of internal diameter) but cannot be used efficiently with plastic pipes whose internal diameter varies a lot and the roughness is limited to three or four values.



For instance, for a DN100 pipe, with a velocity of 1 m/s, the flow is about 8 l/s and the friction should be between 12 and 20 m per km according to the roughness k.

The **Log Scales**: this is a simple and quick way to find the losses or the needed diameter. It is composed of four logarithmic scales representing the internal diameter [mm], the flow [l/s], the velocity [m/s] and the losses [m/100m]. A line can be drawn between any two points and the two other parameters can then be read. It gives a good feeling about the effect on changes but the accuracy is quite bad.

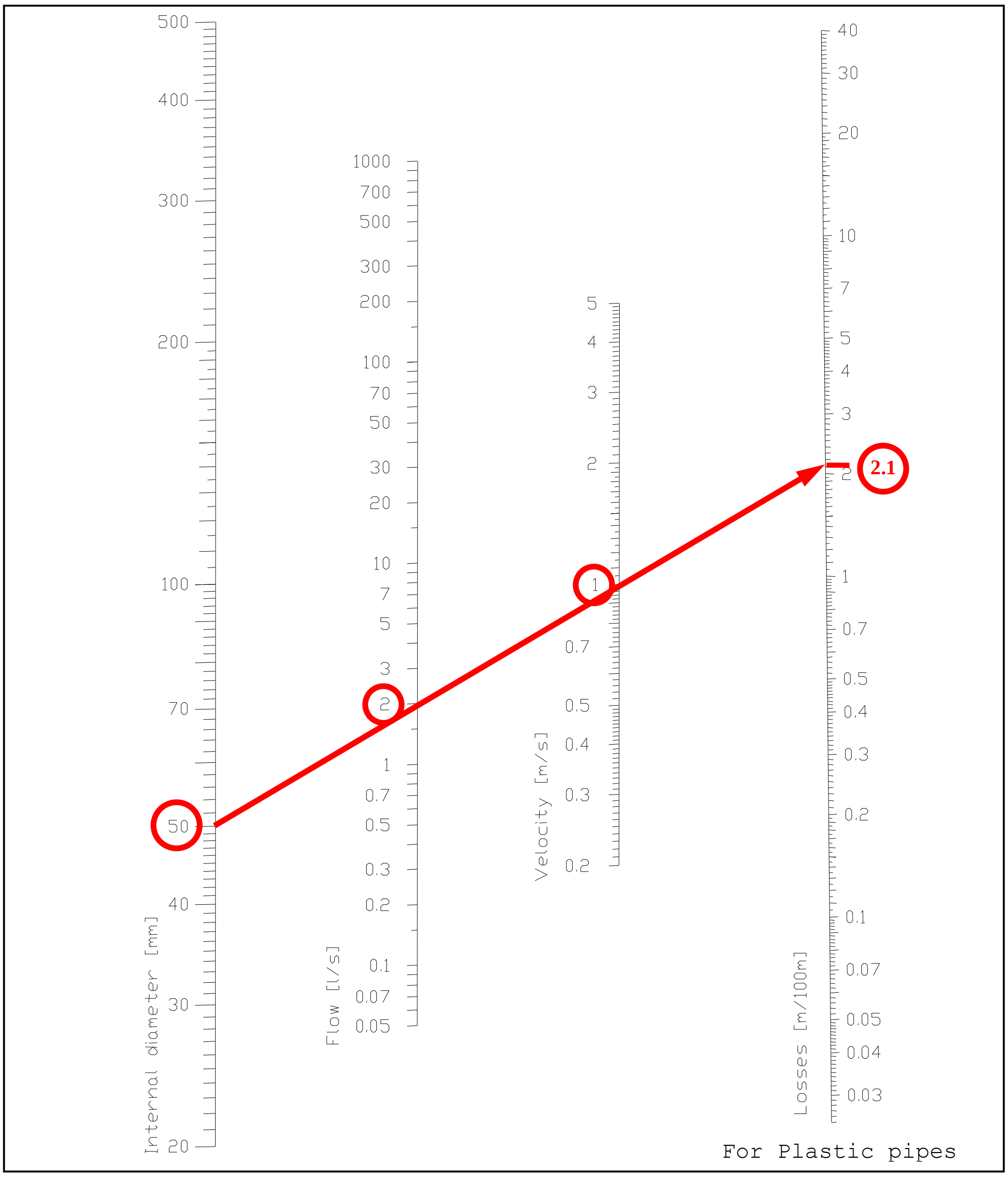


Fig 4- 6 Log scale

For instance with a DN 2" (~50mm) we will have a flow of 2 l/s with losses of 2,1 m per 100m length for a velocity of 1m/s.

The third and last "bush" method is the **charts**.
They are quick to use and give a good feeling of the conditions of the flow (if we are in the middle of the table it is fine, else, the calculation should be reconsidered) and the Reynolds can also be easily found. They are also more precise than the Log Scale. However, they are done for a given roughness and are not precise any more if the roughness differs too much.

Note that the internal diameter for plastic pipes varies a lot with the PN and is therefore not very easy to use.

An example is given with the same values as for the Log Slide Rule: Q=29 m^3/h, and a chart for coated metallic pipes (k=0.12 mm). Similar head loss is found, about 1.4 m per 100m.

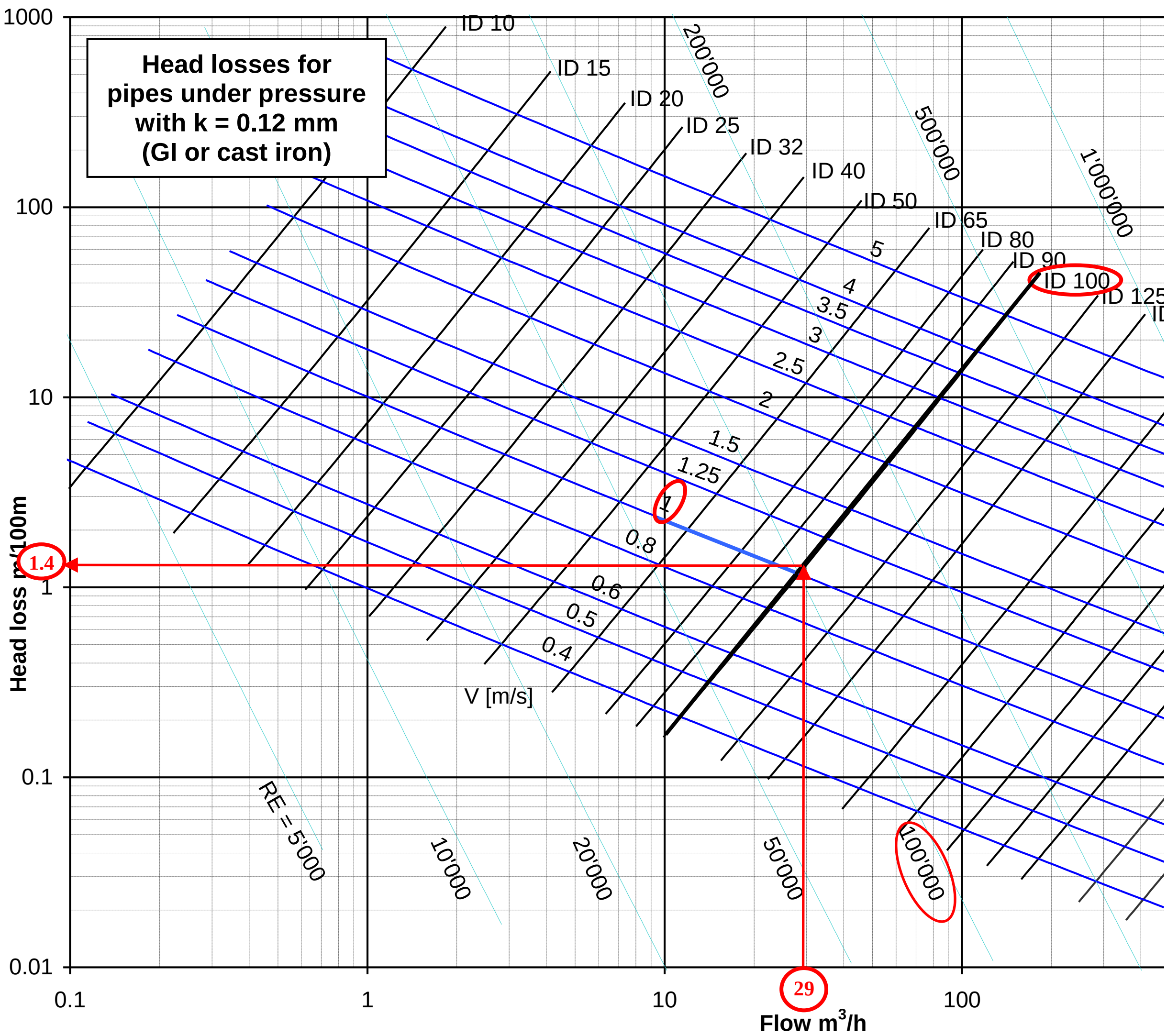


Fig 4- 7 Head losses for pipe under pressure

Charts for other roughness are given in the annexe.

Tables for friction losses coefficients

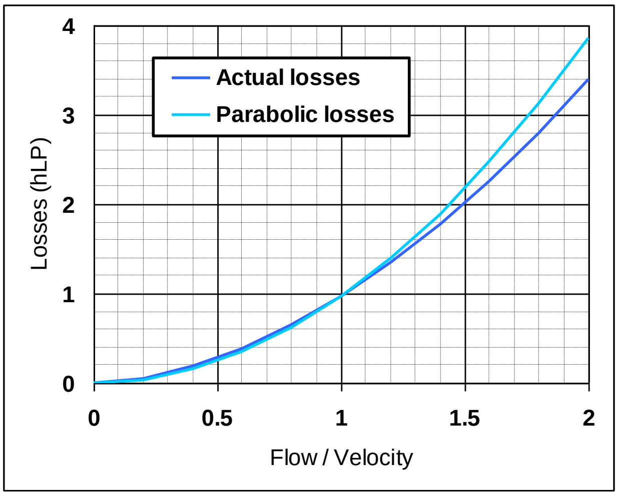
Several documents still give **Tables for friction losses coefficients**, they were done for calculator and should not be used any more as they are not precise at all and give wrong impression of precision.
Computer can easily make all necessary calculations; it will be more precise and much simpler to iterate.

HDPE 25 -ID 20.4mm- PN 16		
J (m/km)	Q (l/s)	v (m/s)
123	1	[illegible]1
456	2	0,2
789	[illegible]	0,3
012	4	0,4
345	5	0,3

Not to be used

4.9. System curves

As it has been shown (eq.4-11), head losses depend on one hand on the system (punctual and linear losses) and on the other hand on the flow. Thus, h_{LP} could be calculated for given water scheme for different flows, and represented in a chart as the example in the side.
The curve is close to a parabolic function but not exactly as the linear losses depends also on the velocity.
In the attached chart, the flow coincides with the velocity. The head losses coefficient used to draw the parabolic curve is the value obtain with a velocity of 1 m/s (in the middle of the curve). As it can be seen, the curves diverge quickly after 1.3.
This concept will be used later to define the working point of a system with pump.



References for this chapter:

KSB : Selecting Centrifugal Pumps

Basic exercises

1. What are the ID, the SDR and the Series of a pipe with an outside diameter of 110 mm and a thickness of 6.6 mm?
2. What is the nominal pressure of this pipe if it is made with PVC and used at a temperature of 30°c? If it is made with PE80, PE100 at 20°C?
3. What is the nominal pressure of these PE pipes if it is used for gas (the service ratio is 2 for gas)?
4. What are the friction losses in a DN150 PVC pipe of 2.2 km, with a velocity of 1 m/s? Same question for a cast iron pipe (k=0.12mm) (can be estimated with figures: either log scale of charts in annexes)?

Intermediary exercises

5. What is the hydraulic diameter of a square pipe (b=h)? Of a flatten (elliptic) pipe with b=2h?
6. What is the minimum velocity and flow that should flow to avoid air pocket (Re = 10'000) in a pipe of DN25, DN200, DN500 ?
7. What is the punctual friction losses coefficient for a pipe connected between to tanks, with four round elbows (d=D), a gate valve, a non-return valve and a filter?
8. What is the average punctual friction losses coefficient for the accessories of a DN200 pump?
9. What are the friction losses in a DN150 PVC pipe of 2.2 km, with a velocity of 1 m/s? Same question for a steel pipe (k=1mm) (to be calculated with equations, not estimated with charts)?
10. What should be the diameter of an orifice to create losses of 20 meters in a DN100 pipe with a velocity of 1 m/s?

Advanced exercises

11. What is the flow in a DN150 steel pipe of 800 m connecting two tanks with a gate valve, five elbows and a filter, if the difference of height between the tanks is 2m; 5m; 10m?
12. How much shall a pipe be crushed to reduce the flow by half?
13. What is the flow when the pipe is crushed to half of its diameter?

Chapter 5. Open flows

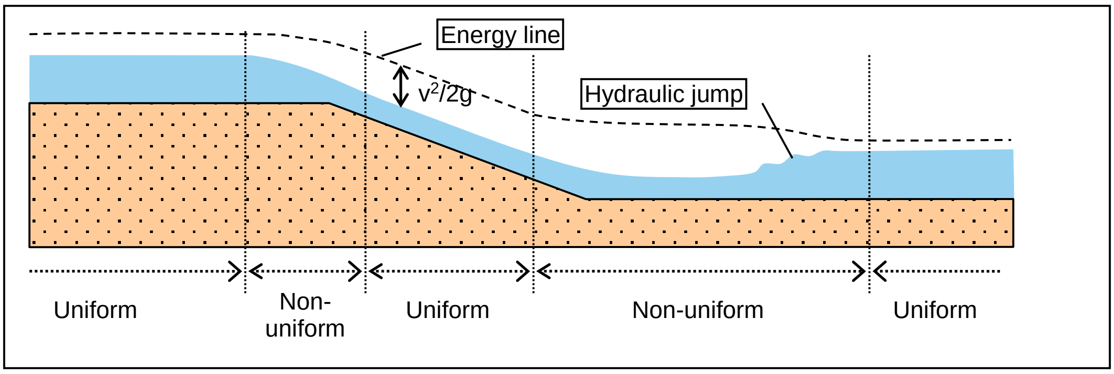
What you will learn
Understand types and specifications of open flows.
Calculate the flow velocity for simple cases
Use weirs to measure the flow

Why
To be able to understand and predict the flow velocity in pipes when they are not full.

Duration of chapter 5
3 to 4 hours

*5.1. Introduction to open flows **

An open flow occurs in a pipe or a duct when the liquid flows with its surface at atmospheric pressure (free surface). In this case, there is only kinetic and potential energy, no "pressure" energy. The flow is then only dependent on the slope and the surface of the water. Contrary to flow in pipe under pressure, the cross-section is not fixed and depth will vary.
The types of open flow are first classified as steady if it is constant with time or unsteady when the flow is started, stopped or when there are waves. The unsteady conditions are complicated and will not be tackled further.
The second classification is as *steady uniform* flow; it will occur over a long portion of constant cross section and slope, in this case, the head losses will be equal to the difference of bed elevation. In between those portions, the flow will be *steady non-uniform*, in which case it might be slow, fast or with a hydraulic jump; the study of the non-uniform condition is mainly useful to design spillways and flumes or other component of canals, it will not be tackled in this chapter.



Uniform and non-uniform steady flows in open channels

As the pressure is always at the same level, it can be deduced from the Bernoulli's equation that the energy line will be $v^2/2g$ above the free surface.
For uniform flows, the energy line will be parallel to the surface of the water and the bed. For non-uniform flows, it will be divergent when the flow is accelerating and convergent when it is slowing down; in hydraulic jump the losses are quite important.

*5.2. Hydraulic radius **

The hydraulic radius is defined as the ratio of the cross-section area to the wetted perimeter (length of contact between the liquid and the channel).

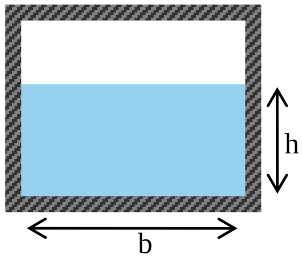
R_h: hydraulic radius
A: section [m^2]
P: wetted perimeter [m]

$$\boxed{R_h = \frac{A}{P}}$$ **Eq. 5-1**

For rectangular sections

b: width [m]
h: height of liquid [m]

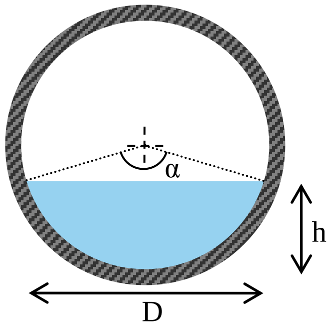
$$\boxed{\begin{aligned} A &= b \cdot h \\ P &= b + 2 \cdot h \end{aligned}}$$ **Eq. 5-2**



For cylindrical sections

D: Pipe diameter [m]
h: height of liquid [m]
α: angle from pipe centre to the surface [rad]
$\alpha_{rad} = \alpha_{deg} \cdot \pi/180$

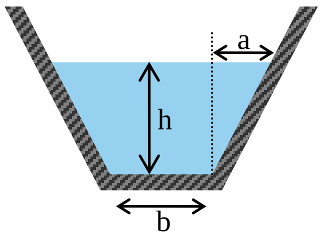
$$\boxed{\begin{aligned} A &= \frac{D^2}{8}\left(\alpha - Sin(\alpha)\right) \\ P &= \frac{\alpha}{2} D = arcCos\left(1 - 2\frac{h}{D}\right) \cdot D \\ \alpha &= 2arcCos\left(1 - 2\frac{h}{D}\right) \end{aligned}}$$ **Eq. 5-3**



For trapezoidal sections

b: bed width [m]
a: additional half width at the surface[m]
h: height of liquid [m]

$$\boxed{\begin{aligned} A &= (b + a) \cdot h \\ P &= b + 2\sqrt{a^2 + h^2} \end{aligned}}$$ **Eq. 5-4**



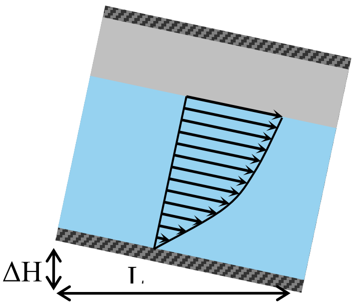
NB: The hydraulic radius is a different notion from the hydraulic diameter studied in the previous chapter, and should not be confused. For an almost full pipe, R_h=D/4.

The optimum shape for a channel is when its hydraulic radius is maximum or thus its wetted perimeter is minimum. It can then be demonstrated that the best proportions of a channel to have the maximum flow is :

For a rectangular channel, the height should be half the width
For a cylindrical pipe, it should be half full
For a trapezoidal channel, it must be a half hexagon

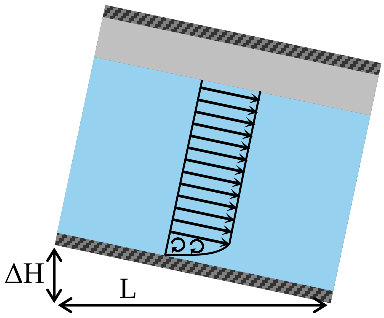
5.3. Reynold's number

As for pipes, the flow can be laminar or turbulent with respectively a parabolic and an exponential profile of velocity. In practice, laminar flow is rare and will only occur if its flow is very shallow.



Laminar flow
Re < 2 000

Slope :
$S = \Delta H/L$



Turbulent flow
Re > 10'000

The Reynolds number can be calculated using the mean velocity of the section; the diameter should be replaced by four times the hydraulic radius (also called hydraulic mean depth).

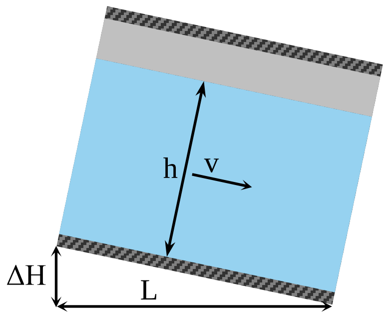
- Re: Reynolds number
- R_h: hydraulic radius [m]
- v: mean velocity of the section [m/s]
- ν: kinematic viscosity

$$\boxed{Re = \frac{4 \cdot R_h \cdot v}{\nu}}$$ **Eq. 5-5**

5.4. Manning's Equation *

For steady uniform flow, the head losses are equivalent to difference of height. Thus, the velocity is defined by the slope, the type of material of the duct and hydraulic radius. This relation is given by the **Chezy formula**

The coefficient has not an exact value; it was estimated by different people by experience (Kutter, Bazin, Powell, Darcy, Manning).



- v: Mean velocity [m/s]
- C: Coefficient
- R_h: Hydraulic radius [m]
- S: Slope $\Delta H/L$ [-]

$$\boxed{v = C \cdot \sqrt{R_h \cdot S}}$$ **Eq. 5-6**

For laminar flow, as for pipe under pressure, the roughness has no influence on the losses, the coefficient can be estimated as :

$$\boxed{C = 1.107\sqrt{Re}}$$ **Eq. 5-7**

NB as explained in the introduction, open flows are rarely laminar.

For turbulent flow, the **Manning coefficient** will be used, it is a given by the ratio of the hydraulic radius and the Manning factor given in the table below:

$$\boxed{C = \frac{R_h^{1/6}}{n}}$$ **Eq. 5-8**

With equations 5-6 and 5-8 the Manning's equation is given as follow:

- v: Mean velocity [m/s]
- R_h: Hydraulic radius [m]
- S: Slope $\Delta H/L$ [-]
- n: Manning's coefficient

$$\boxed{v = \frac{R_h^{2/3} \cdot S^{1/2}}{n}}$$ **Eq. 5-9**

Table 5- 1 Manning coefficient

Surface of channel / pipe	**Good**	**Poor**
Neat cement	0.0106	0.013
Cement mortar	0.011	0.015
Concrete	0.012	0.018
Cast iron	0.013	0.017
Plastic pipes	0.0106	0.012
Canals earth	0.017	0.025
rock cuts	0.025	0.035
Natural streams clean & smooth	0.025	0.035
rough	0.045	0.060
very weedy	0.075	0.15

NB The value of n=0.0106 correspond to a smooth surface and is the minimum that can be used.

Resolution of the Manning's Equation: In most of our application, the flow is known as the velocity and the height of liquid is searched. The velocity is then deduced from the flow and the section; in this case, the height of liquid appears in both side of the equation and is not solvable easily mathematically. The quickest way to find the height is by iterative approximations (see exercises) or using Excel with the function "Goal Seek".

5.5. Pipes partially filled

To ease the use of Manning's equation with pipes in practical situations, two "manual bush methodology" were developed for the two main cases where pipes might not be under pressure:

A. **sewer or drainage** pipes where the **slope** is fixed but the **flow** varies
B. **gravity fed water** pipes where the **flow** is fixed but the **slope** varies

In both cases, the first step is to define for a given diameter the conditions for which the pipe is full with the following chart.

A. with the known slope define the velocity and flow at full condition (v_f and Q_f).
B. with the known flow define the velocity and slope at full condition (v_f and S_f).

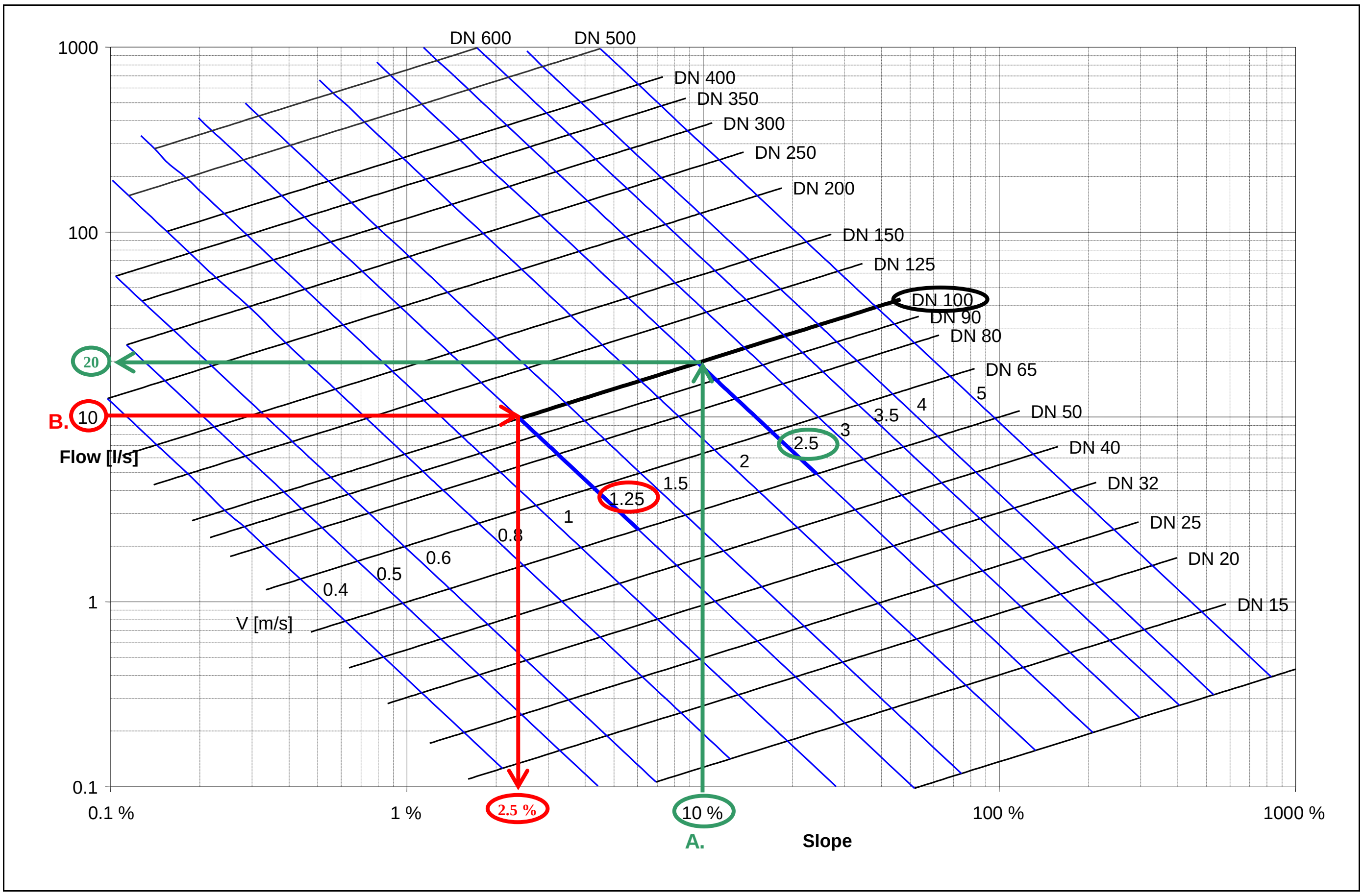


Fig 5- 1 Velocity and flow for a given slope and diameter at full condition.

NB This chart is only valid for smooth pipes (Manning coefficient of 0.0106); as for the previous chapter, the drawback of the charts is that they are only valid for a given roughness. Charts for other coefficients are given in the annexes.

The following examples treat the two cases with a DN100 (internal diameter) pipe.

A. With a slope of 10%, it will be full at a flow of Q_f=20 l/s with a velocity of v_f=2.5 m/s.
B. With a flow of 10 l/s, it will be full at a slope of S_f=2.5% with a velocity of v_f=1.25 m/s.

A. In the first case (fixed slope), we want to know how the filling and velocity varies with the flow. Therefore, the flow ratio (actual flow divided by flow at full condition) should be calculated, then the filling and velocity ratio can be read in the following chart.

In this example with fixed slope, if the flow is bigger than 20 l/s, the pipe will be under pressure, if the flow is smaller then the pipe will be partially filled.

For instance with an actual flow Q=12 l/s (DN100, S=10%, Q_f=20 l/s and v_f=2.5 m/s):

a) $Q_\% = Q / Q_f = 12 / 20 = 60\%$
b) With the flow curve $h_\% = 56\%$
c) $h = h_\% \cdot DN = 0.56 \times 100 = 56$ mm
d) With the velocity curve $v_\% = 105\%$
e) $v = v_\% \cdot v_f = 1.05 \times 2.5 = 2.63$ m/s

Thus, the height of water will be 66mm at a velocity of 2.63 m/s.

This chart shows also that velocity in the pipe is the same when it is full or half-full and the highest velocity is around 80%.

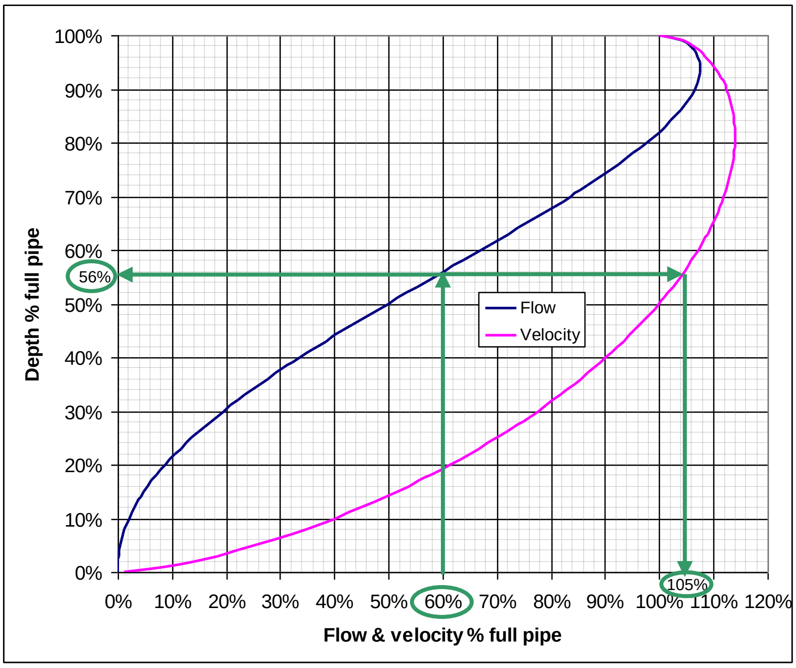


Fig 5- 2 Flow and velocity in a pipe according to its filling

It can also be noticed that between 82% and 100% of the depth, the flow can even be larger than when it is full. However, this zone has an instable behaviour and should be avoided for a pipe that cannot bear pressure. Therefore, sewerage pipe should never be more than 80% full.

B. In the second case (fixed flow) we want to know how the filling and velocity varies with the slope. Therefore, the slope rate (actual slope divided by slope at full condition) should be calculated, then the filling and velocity rate can be read in the attached chart.

In this example with fixed flow, if the slope is less than 2.5% then the pipe will be under pressure; if it is steeper, the pipe will be partially filled.

For instance with an actual slope S=20% (DN100, Q=10 l/s ,S_f=2.5%, and v_f=1.25 m/s):

a) $S_x = S / S_f = 20 / 2.5 = 8x$
b) With the velocity curve $v_x = 2.6x$
c) $v = v_x \cdot v_f = 2.6 \times 1.25 = 3.25$ m/s
d) With the height curve $h_\% = 41\%$
e) $h = h_\% \cdot DN = 0.41 \times 100 = 41$ mm

Thus, the height of water will be 41mm at a velocity of 3.25 m/s, which might be already too high if there is sand in the water (problem of erosion).

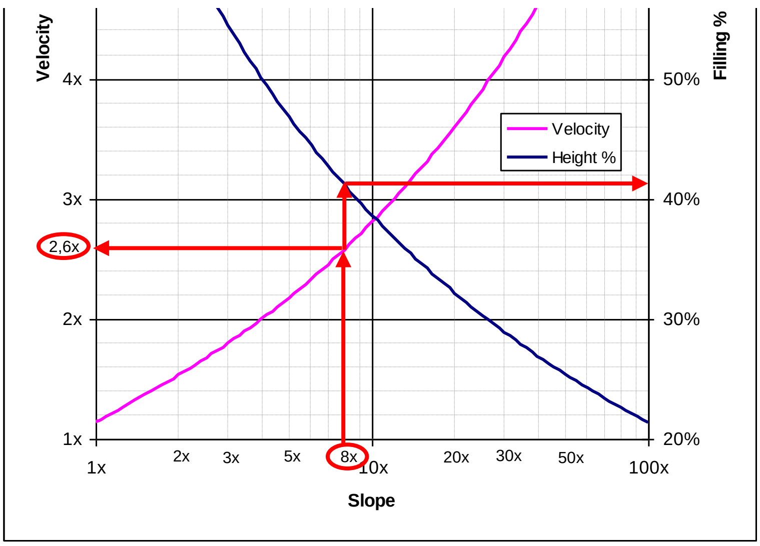


Fig 5- 3 Velocity and depth in a pipe according to its slope

For gravity systems supplied with a limited flow, large parts of the pipes might be partially filled. In this case it is important to check that the velocity is not too high, else the pipe might be eroded. Maximum acceptable velocity is usually taken between 3 to 5 m/s according to the abrasive properties of water.

5.6. Weirs

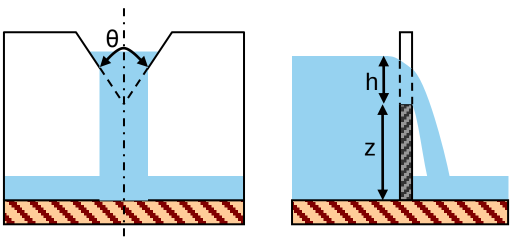
A simple and precise way to measure the flow in channels, collection boxes or out of a tank is to use a weir. The triangular or V notch shape is good for small and variable flow, as the rectangular one is for flow rates greater. To ensure a good measure, the waterfall should be complete so that the downstream cannot affect the upstream level.

For a triangular weir with a thin crest :

Q: Flow [m^3/s]
c: Coefficient
θ: Notch angle [°]
h: Height of water [m]

$$Q = \frac{4}{5} c \cdot tg\left(\frac{\theta}{2}\right)\sqrt{2g} \cdot h^{5/2}$$ Eq. 5-10

The c coefficient depends on the type of flow, it is usually around 0.4 for a turbulent flow and Z >> h. To determine it precisely, it should be measured as it depends on the thickness, the contraction ratio, the precision of the angle and the type of flow.
A bump on the curve is visible at the passage from laminar to turbulent condition.



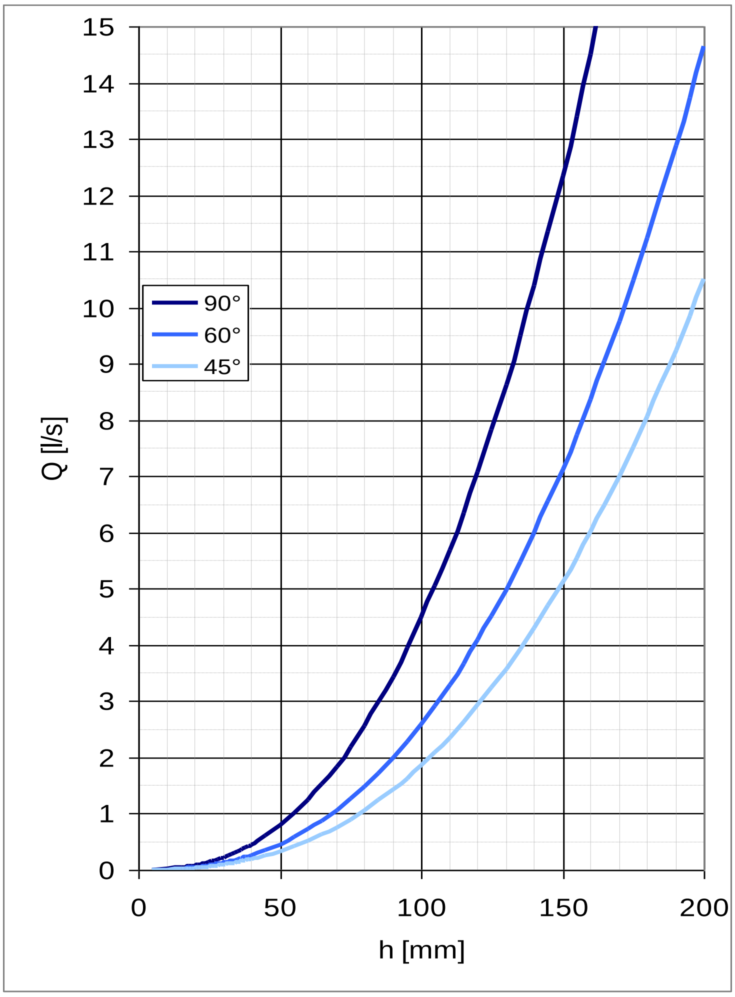


Fig 5- 4 Flow through V-notch weirs

In figure 5-4 we can see the theoretical curve for V-notch weirs of 45°, 60° and 90° with c=0.4. V-notch with small angle should be used for small flow.
A precise V-notch can easily be house made with a rigid plastic sheet, but needs to be calibrated to have the correct h / Q curve.

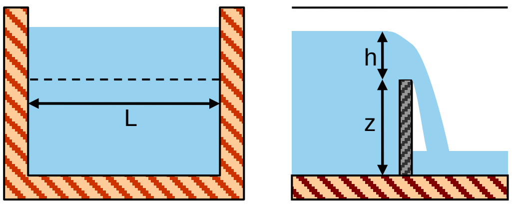
For a rectangular weir without side contraction: the flow is defined as follow:
Limit conditions: 0.025m<h<0.8[m]; z>0.3 [m] and h<z

Q: Flow [m^3/s]
c: Coefficient
L: Width [m]
h: Height of water [m]
z: Height of crest [m]

$$\boxed{Q = c \cdot L \cdot \sqrt{2g} \cdot h^{5/2}} \quad \textbf{Eq. 5-11}$$

$$\boxed{c = 0.410 \cdot \left(1 + \frac{1}{1000h + 1.6}\right) \cdot \left(1 + 0.5\left(\frac{h}{h+z}\right)^2\right)} \quad \textbf{Eq. 5-12}$$

When the approach velocity is small, the coefficient c is usually around 0.4.



Basic exercises

1. What is the hydraulic radius of a square channel with the same width as depth (h=b)?
2. What is the hydraulic radius of a half-full pipe (h=D/2), of a full pipe (h=D)?
3. What is the flow going through a rectangular channel 6m wide, 1m deep with a slope of 0.0001 (n=0.015)?
4. What is the depth of water in a rectangular channel 6m wide with a slope of 0.0001 and a flow of 6m3/s (n=0.015) ?
5. What is the width of a rectangular channel to carry 13.5m^3/s with a 1.8 m water depth and a slope of 0.0004 (n=0.012) ?

Intermediary exercises

6. What is the minimum flow to have in a channel 6m wide and 1.5m water depth to be sure that the flow is turbulent?
7. A cylindrical pipe with a slope of 0.002 should carry 2.30m^3/s, we want it to be 80% filled. What should be the diameter (to be done with the equations)?
8. What are the velocity and the height in a DN100 pipe with a flow of 20l/s and a slope of 40% (to be done with the fig. 5-1 & 5-2)?
9. What is the maximum acceptable slope for a smooth DN100 pipe with a flow of 10 l/s, if we want to limit the velocity at 3m/s; at 5m/s (to be done with the fig. 5-1 & 5-3)?
10. What is the flow going through a V notch weir of 60° with the height of water of 10cm?

Advanced exercises

11. Show that the best hydraulic section for a rectangular channel is b=h/2.
12. Knowing that the best hydraulic section for a trapezoidal section is a half hexagon, calculate b, a and h for a trapezoidal channel having an area of 4m^2?
13. What is the flow going through a rectangular weir of 2m width, with a crest height of 1m and a height of water of 50cm?

Chapter 6. Hydraulic pumps

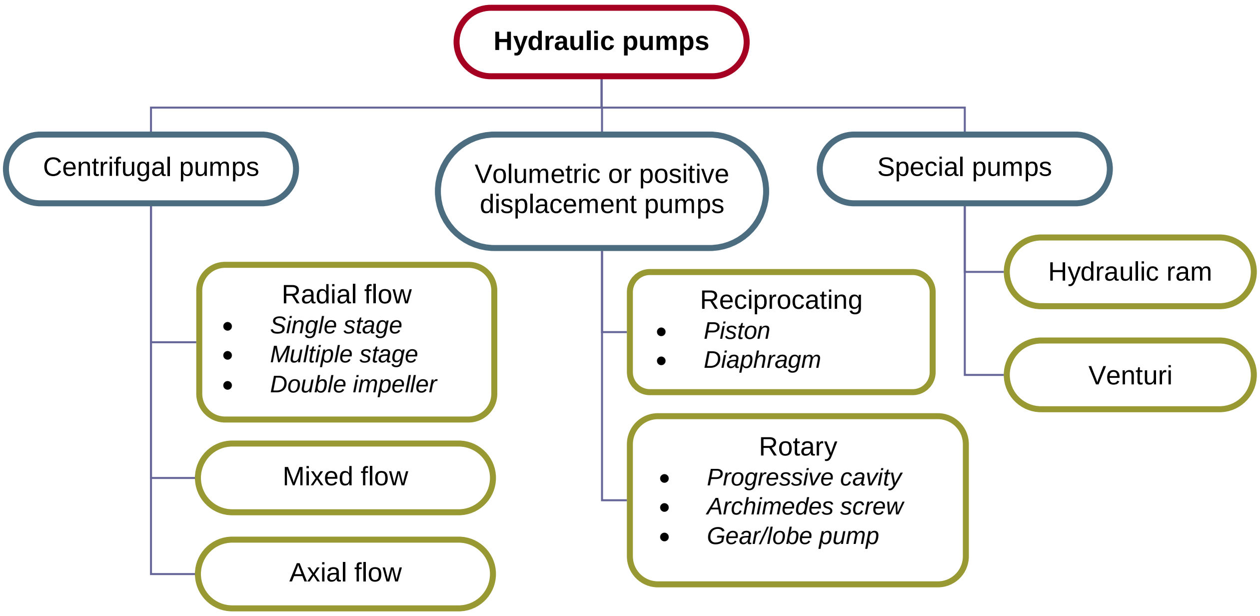
What you will learn
Understand the basic on hydraulic pumps and the specific way how the centrifugal pumps are working.

Why
Centrifugal pumps are an essential part of most of water supply systems. The good understanding of their characteristics and functioning is essential to assure that the best option is selected and ensure efficiency and sustainability.

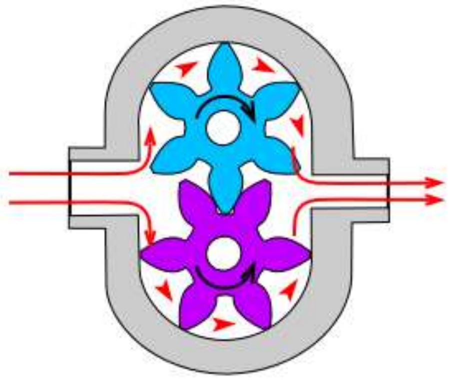
Duration of chapter 6
3 to 4 hours

*6.1. Types of hydraulic pumps **

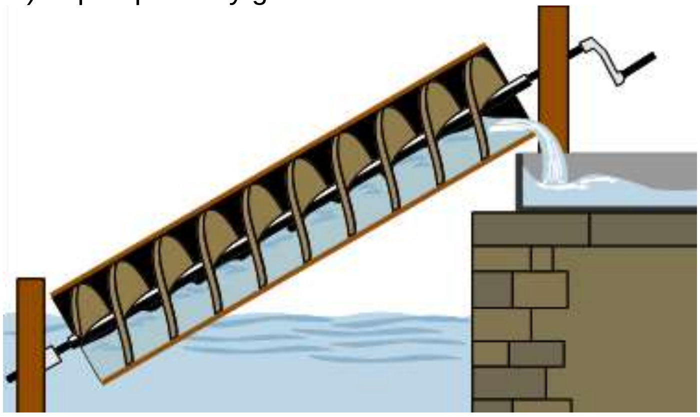
The role of a pump is to transform mechanical power into hydraulic power by increasing the pressure of the pumped fluid or forcing a flow. A huge variety of pumps exists for a wide range of specific applications. They are usually subdivided in three categories, the centrifugal pumps are kinetic machine in which energy is continuously transmitted to the fluid by a rotating impeller, the positive displacement pump moves the fluids by trapping a fixed amount and forcing it mechanically into the discharge and the special pumps are the types not fitting in the two first categories.



In water systems, most of the pumps used for transport are centrifugal (with the exception of rotary progressive pumps used for low flow and relatively high head specification like borehole pumping with solar power) and for dosing (in treatment) are reciprocating pumps (piston or diaphragm depending on the fluid) where a small accurate flow is needed at a high pressure. Special pumps include the hydraulic ram, which is a machine working without motor, using the energy of a part of the water, released at the pump level, to raise the rest of the water at a higher level; and the Venturi pump using the Venturi effect (see §3.2) to pump mainly gas.

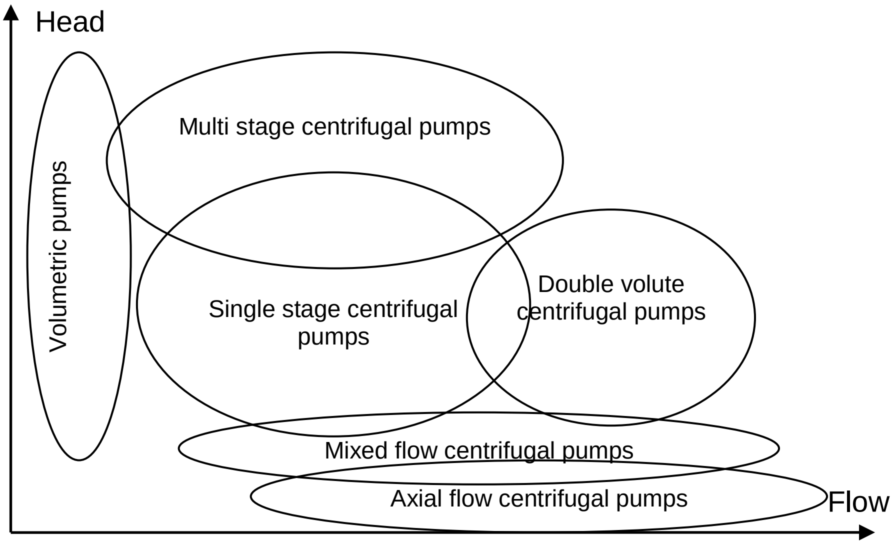


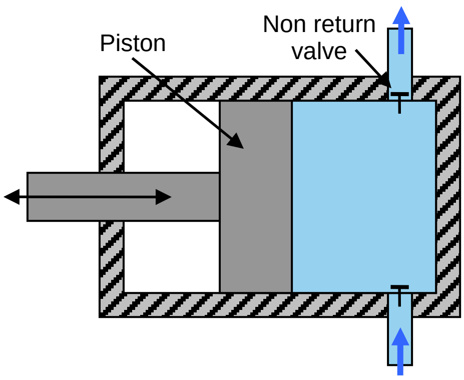
Gear pump



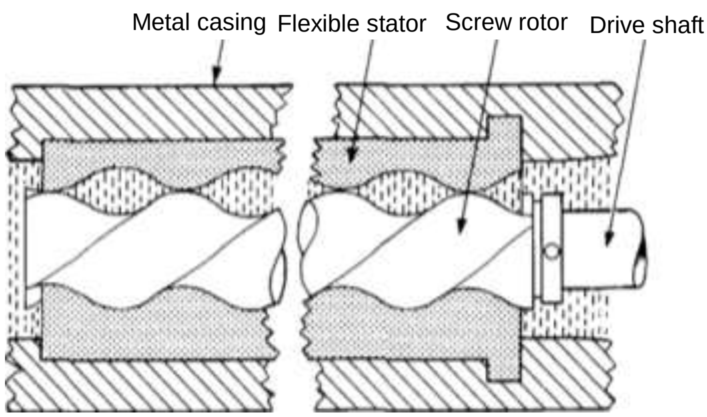
Archimedes screw

Centrifugal pumps are usually divided in three groups characterized by their shape (radial, mixed and axial flow), the radial are themselves subdivided in three groups (single stage, multistage and double impeller or volute) defining their working range. The following figure represent the range were those different pumps are used.





Piston pump



Progressive cavity pump

In some cases, additional characteristics should be specified such as:

- Position of the shaft (horizontal or vertical)
- Location of the pump (dry or submersible)
- Closed or open impeller
- Type of liquid to be pump (clean water, sewage, abrasive…)
- For boreholes (with specifically small diameter)

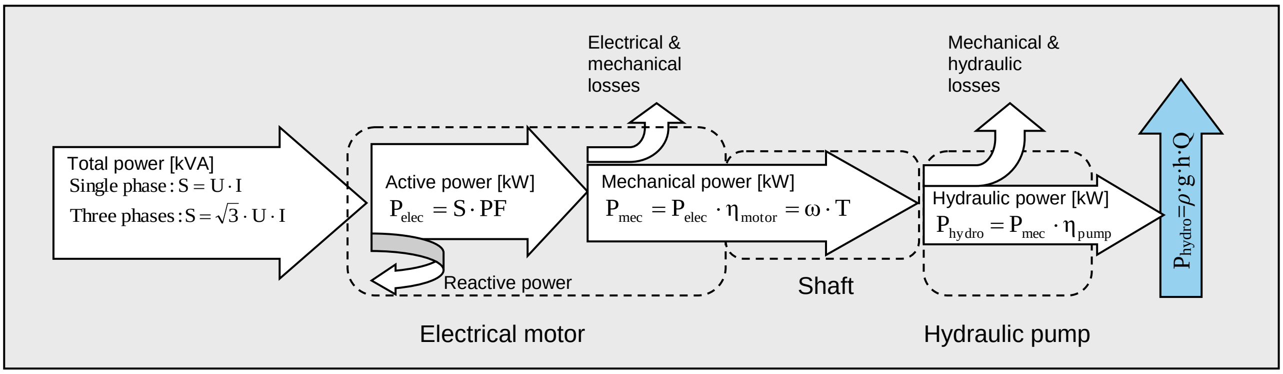
- Position in the system (suction, boosting)
- Number of poles (2,4 or 6 defining the rotation speed)
- Type of seal (mechanical or packing)
- Self priming
- Etc

In this chapter we will first study the power and efficiency, a common aspect for all pumps, and then we will deepen mainly the cases of centrifugal pumps as they are the most common pumps for water supply and have quite special characteristics depending on many factors correlated.

6.2. Power and efficiency *

The pump will transform the mechanical power into hydraulic power by increasing the pressure of a certain quantity of water. This increase of pressure is known as the manometric head, or simply the head of the pump written h (expressed in meter). This mechanical power is usually provided by an electrical motor but can also come directly from human action (hand pump), from the wind or combustion engine. This chapter will mainly focus on the pump driven by electrical motor, but similarity can be done for combustion engine; however, the efficiency is much lower.

The next figure represents the flow of energy (power) with the different losses from the electrical motor through the shaft to the pump where it "powers" the water.



The transmitted power is decreased at each step and can be listed as follow from the biggest to the smallest:

- **Electrical power**: is the electrical power consumed by the motor.

S: total (apparent) power [VA]
U: electrical tension [V]
I: electrical intensity [A]
P_{elec}: active (true) power [W]
PF: power factor or cos phi

$$\boxed{\begin{aligned}&\text{Single phase}: S = U \cdot I \\ &\text{Three phases}: S = \sqrt{3} \cdot U \cdot I \\ &P_{elec} = S \cdot PF = S \cdot \cos(\varphi)\end{aligned}} \quad \textbf{Eq. 6-1}$$

- **Mechanical power**: is the power transferred from the motor to the pump by the shaft.

P_{mec}: mechanical power [W]
η_m: motor efficiency [-]
ω : rotation speed (rad/s)
T : Torque [N/m]

$$\boxed{P_{mec} = P_{elec} \cdot \eta_m = \omega \cdot T} \quad \textbf{Eq. 6-2}$$

- **Hydraulic power**: is the power transmitted to the water in the pump.

P_{hdro}: hydraulic power [W] η_p: pump efficiency [-] ρ: water density [kg/m3] g: earth gravity [m2/s] h: manometric head [m] Q: flow [m3/s]	$P_{hydro} = P_{mec} \cdot \eta_p = \rho \cdot g \cdot h \cdot Q$ $P_{mec} = \dfrac{\rho \cdot g \cdot h \cdot Q}{\eta_p}$	**Eq. 6-3**

Thus, the whole system can be written as in the next equation, allowing estimating the necessary current. NB According to Eq. 6-1 the $\sqrt{3}$ should be removed for single-phase motors.	$S = \sqrt{3} \cdot U \cdot I = \dfrac{\rho \cdot g \cdot h \cdot Q}{PF \cdot \eta_m \cdot \eta_p}$	**Eq. 6-4**

Rough evaluation of the pump and motor efficiency and power factor:

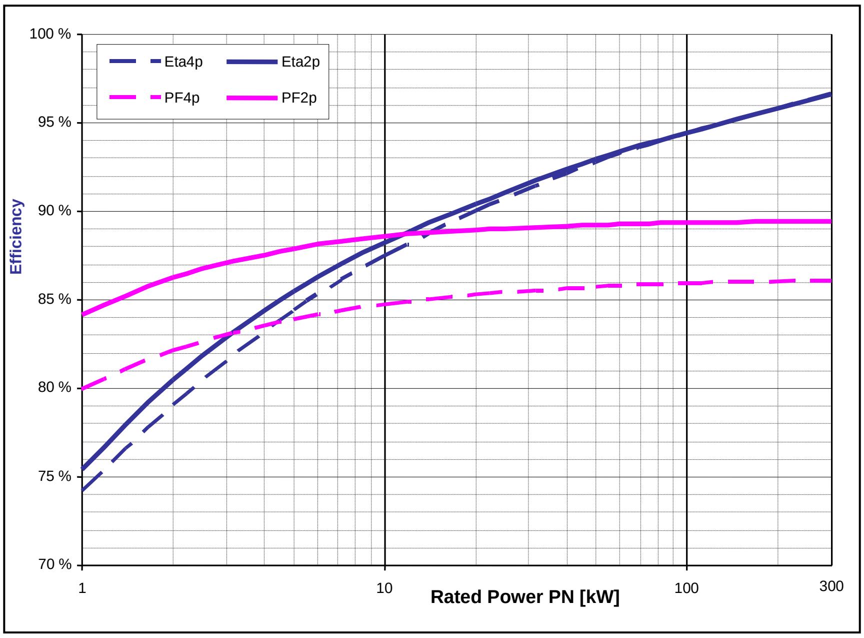


Fig 6- 1 Efficiency versus rated power

For electrical motor:

The efficiency of a motor at its rated power mainly depends on its size and number of poles (2p or 4p, cf §6.3).
The power factor (PF) varies according to the same values.

In the attached chart expectable efficiency for commercial motors are given according to these two values.

It is possible to ask for higher efficiency motors, but they are more expensive especially for small power.

For centrifugal pumps:

The efficiency at the optimal working point of a centrifugal pump varies a lot. It mainly depends on the flow and the specific speed (cf. §6.5).

In the attached chart expectable efficiency for commercial pumps are given according to these two values.

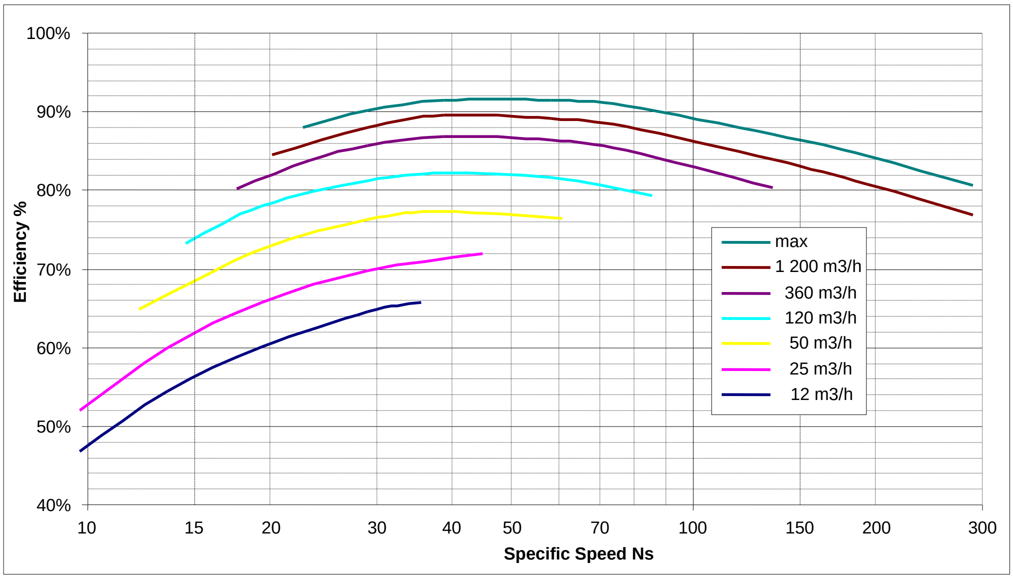


Fig 6- 2 Efficiency versus specific speed

6.3. Electrical motor rotation speed

The big majority of electrical motors used to pump water are of asynchronous type (induction or squirrel cage). A magnetic field is rotating in the stator at a speed depending on the number of pair of poles and frequency of the power (50 or 60 Hz), this is called the synchronous speed. The rotor (rotating part of the motor) is driven at a slightly lower speed depending on the torque applied on it. The difference between the actual rotation speed and the synchronous speed is called the slip and is usually quite small, from 2% for big motors to 10% for small motors (single phase). The rotation speed for motor is measured in rotation per minutes and can be calculated according to the following equation:

n: rotation speed [RPM]
f: frequency of power [Hz]
nbPoles: number of pair of poles
slip in RPM

$$\boxed{n = 60 \cdot \frac{2 \cdot f}{nbPoles} - slip}$$

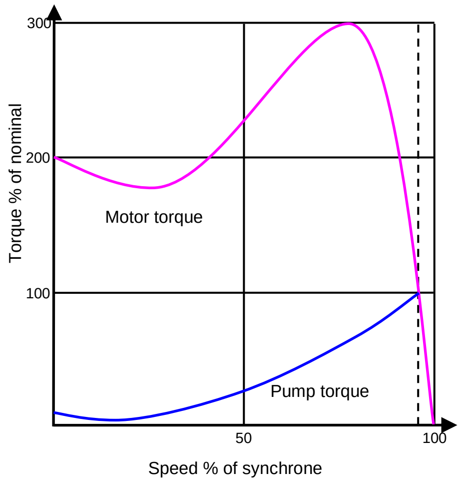
Eq. 6-5

n [RPM]	**Number pair of poles**		
Frequency	2	4	6
50 Hz	2 900	1 450	960
60 Hz	3 500	1 750	1 160

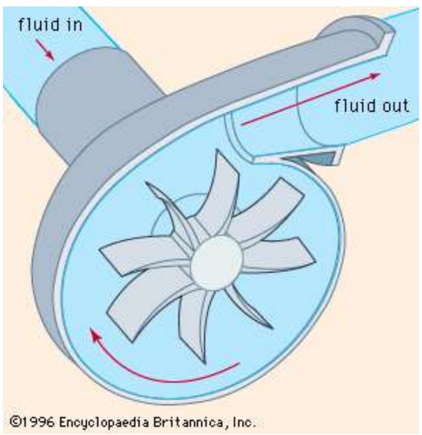
The torque provided by a motor varies according to its speed. A stopped motor (3 phase squirrel cage) will have a starting torque of twice its nominal torque. It will then decrease a bit and then, it will rise up to a maximum called "breakdown torque" around three times its nominal torque. After that, it has almost a linear behaviour between the torque and the slip up to the synchronous speed where the rotor turns as fast as the magnetic field in the stator.
Therefore the rotation speed of a motor will slightly change according to the load, for a motor with a nominal speed of 2900 rpm the following speed can be expected :

Load	0%	50%	100%	200%
Speed	≈3 000	2 950	2 900	2 800

NB if the torque is higher than its nominal value for more than a short time, the motor will start to overheat and will burn after a while. This can be checked with its actual speed if it is lower than its nominal speed.



6.4. Characteristics of centrifugal pumps *



In a (radial) centrifugal pump, the water is introduced at the centre of a chamber in which an impeller is giving a rotating movement to the water forcing it to the periphery of the chamber and increasing its pressure. There, the water is collected in a volute and then through the outlet.
The larger the impeller is or faster it turns, the higher will be the pressure. The flow going through the pump will change dramatically according to the difference of pressure between the inlet and the outlet. This difference of pressure is called the head of the pump.

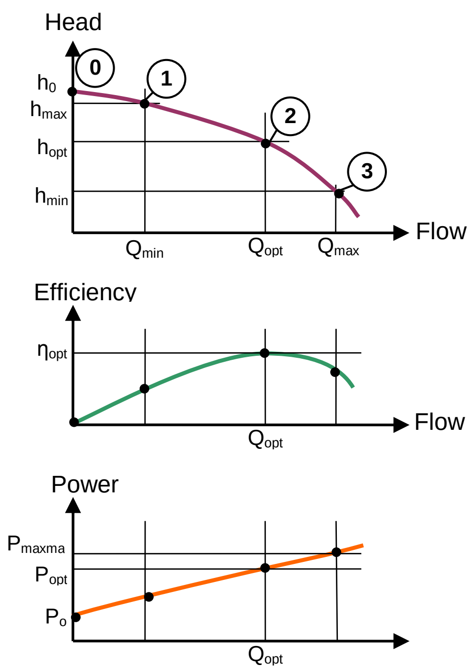
Head versus flow :

Below, the typical behaviour for a radial centrifugal pump is described. For axial pump see § 6.5.

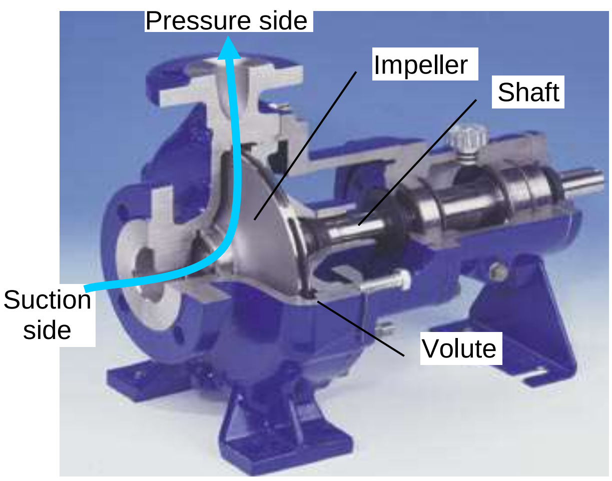
0 A specific pump can increase the pressure only of a certain value (h_0), once this value reached, the water will not move out any more and it will work as a "washing machine" (water is staying inside the pump). If the pump works too long in such conditions, the water will heat and may damage the pump. As there is no flow, the efficiency is nil; the power consumption is at its minimum.

1 If the head is decreased, rapidly the flow will increase to reach conditions where it is safe to run the pump; however, it will still be with low efficiency. This point is usually given as the Q_{min}, h_{max} point.

2 By further decreasing the pressure, the flow will continue to increase as well as the efficiency, until it reaches the optimal point (Q_{opt}, h_{opt}, η_{opt}, P_{opt}) where the water path is well aligned with the blades of the impeller, this is the working point for which the pump was designed.



HQ characteristic of radial centrifugal pump

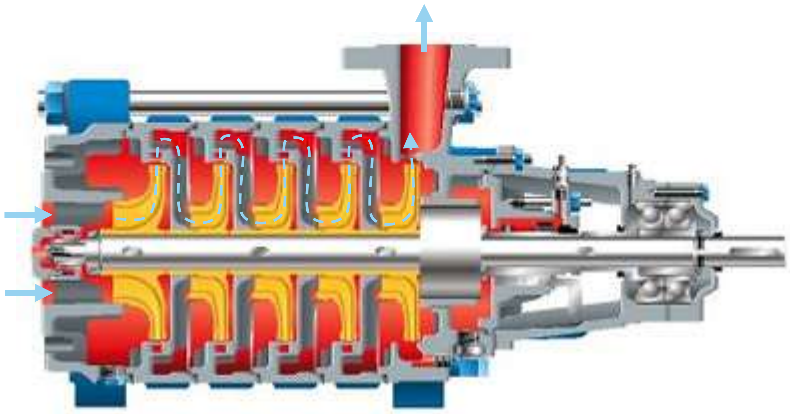


Single stage pump (Etanorm © KSB)

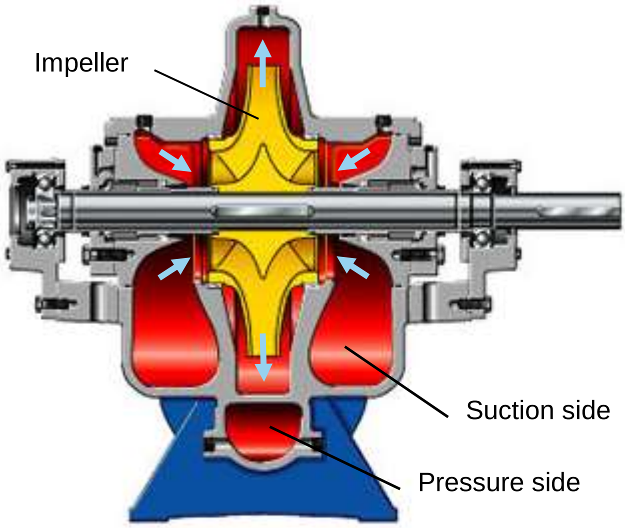
3 Decreasing further the pressure, the flow will still increase but not as fast as before the optimal point, the efficiency will start to decrease, until the maximum permissible flow and minimum permissible head is reached Q_{max}, h_{min} The power request is at its highest level.

Maximum power demand will be at the lowest head and biggest flow

Consequences of having a smaller head might be quite serious and will be explained in § 6.9.



Multi stage pump
(5 stages Multitec © KSB)



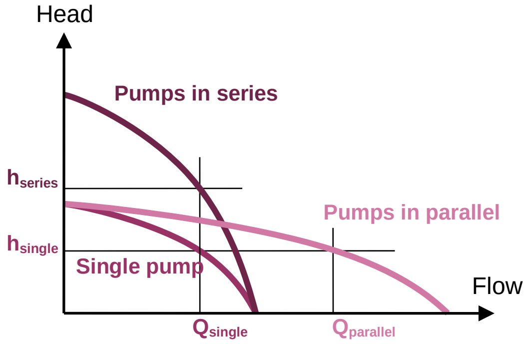
Double volute pump (Omega © KSB)

If a high head is needed, several impellers can be placed in series, summing there characteristics in the same way as pumps in series.

If a large flow is needed, two impellers can be placed in "mirror" two double the flow with the same head in the same way as two pumps in parallel

When centrifugal pumps are connected in series, the head is added for a given flow as represented in the next graph. The efficiency is the average of the initial ones. It is preferable to put in series pumps with the same nominal flow to be able to let both of them work at the optimum flow.

When centrifugal pumps are connected in parallel, the flow is then added for a given head as represented in the next graph. It is preferable to put in parallel pumps with the same nominal head to be able to let both of them work at the optimum flow.

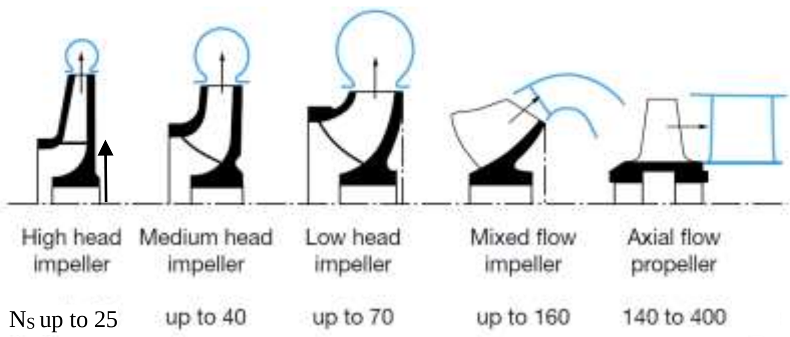


6.5. Specific speed (N_S)

Impellers of centrifugal pumps can be classified according to their proportion. Two impellers with similar proportions but different sizes will have similar characteristics. This proportion is expressed thanks to a number called *Specific speed* (N_S) and defined as per equation 6-6.

$$\boxed{N_S = n \cdot \frac{(Q)^{0.5}}{(h)^{0.75}}} \quad \textbf{Eq. 6-6}$$

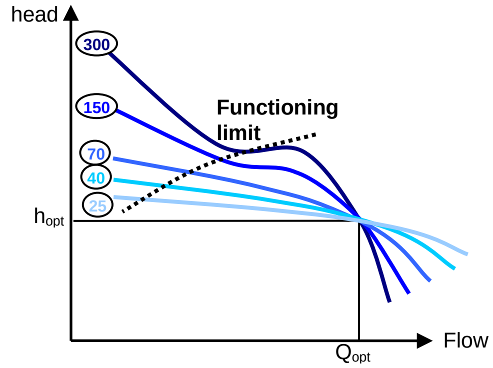
N_S: Specific speed
n: rotation speed [rpm]
Q: nominal flow [m^3/s]
h: nominal head [m]



NB the specific speed should always be calculated for one impeller. In case of multiple-stage pump, the head should be divided by the number of stages and in case of double-volute pumps, the flow has to be divided by two.

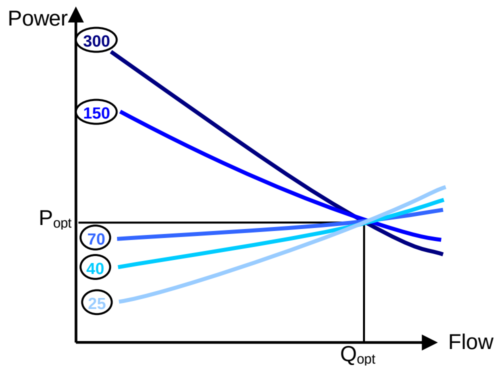
The following charts show the comparison of characteristics of pump with different specific speed.

High head impellers (N_S up to 25) have their outlet diameters more than 4 times bigger than their inlet diameters and have a narrow "channel" (where the water is flowing inside the impeller). Their efficiency are quite bad (cf §6.2) as there is many friction losses in the narrow channel, but the efficiency curves is quite round; it decreases not too quickly around the nominal point. Their HQ curves are very flat, making them interesting to keep a constant pressure with different flow, as in a distribution system – for example for a boosting pump for a house, its maximum pressure is limited and will not decrease too much when many taps are open.



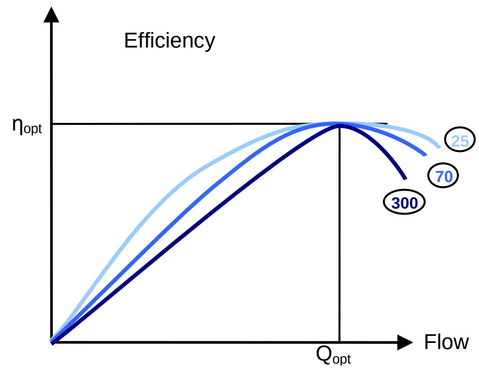
The H-Q curve, is very flat for small N_S and become steeper for bigger one.

Low head impellers or Francis vanes (N_S up to 70) still have a radial operation (water going out of the impeller perpendicularly to the axe) but the outlet diameter is only 1.5 to 2 times the inlet diameter. Its large channel reduces the friction losses giving a very good efficiency. The HQ curve becomes steeper making it interesting to keep a steady flow with head variations as in transport system – for example, a borehole pump, where the level might change with the seasons but the flow should be as steady as possible.



The P-Q curve is increasing for radial pump and decreasing for mixed and axial flow pumps.

Axial flow impellers have an axial operation, pushing the water as a motor boat propeller, their outlet and inlet diameter are equal. This working mode increases the sensibility to hydraulic losses (vortexes) decreasing also slowly the efficiency. This sensibility reduces also the working range and the efficiency curve is quite sharp, it decreases quickly around the nominal point. Problems of cavitation and vortex appear for high head and low head. Unlike the radial pump, the axial pumps will demand less power for a big flow than for a small one.



The η-Q curve is round for small N_S and become sharper for bigger one.

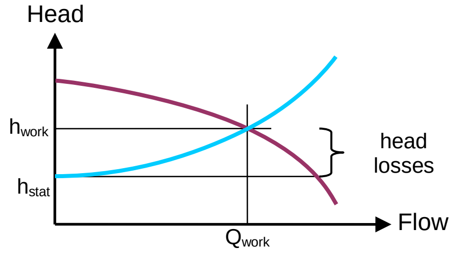
Mixed flow impellers are a mix between the axial and the radial pumps, with properties in between as it can be seen for the curves with a N_S of 150.

For a given head and flow, lower specific speed pump will be larger and more expensive than higher specific speed pump, but cavitation, vibrations and wears will be lessened.

Action	Effect	Result on the N_s
Use 1450 instead of 2900 rpm motor	Reducing the velocity	Divided by 2
Use double volute (impeller) pump	Reduce flow per impeller	Reduced by 30%
Use two stages pump	Reduce head per impeller	Increased by 68%
Trim an impeller	Reduce head & flow	Increasing slightly

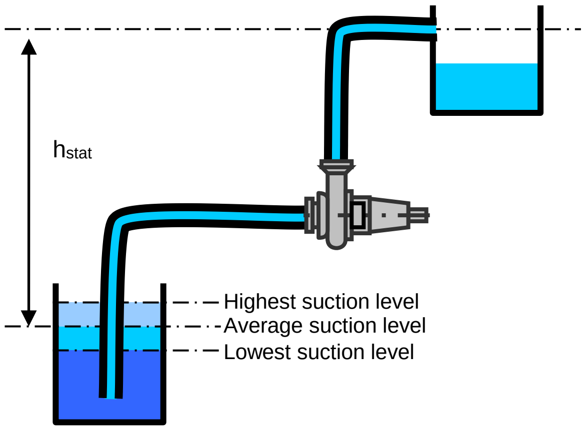
6.6. *Defining the duty point and working range **

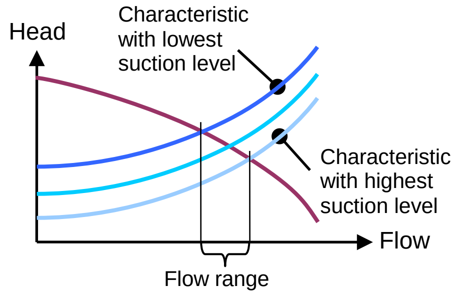
As the pump, the water system (pipes & fittings) on which the pump is working has its HQ characteristic; the working point will be the intersection of these two characteristics as shown in the next chart.
The system characteristic is composed of the addition of the static head and the head losses for the different flows:
The **static head** is the difference of altitude in meter between the average level of the downstream and upstream free water surface. On the suction side, the length of the pipe below the water level should not be taken into consideration.
The **head losses** depend on all linear and punctual losses. In this calculation, the length of the pipe below the water has to be taken into consideration and calculated as seen in chapter 4.



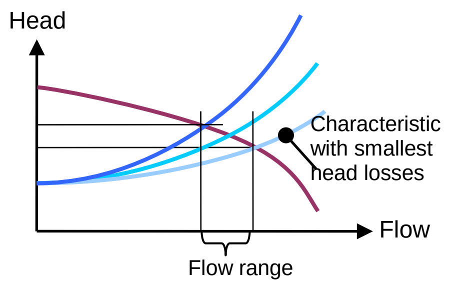
Working point is the intersection between water system and pump characteristics.

The hydraulic condition of the system might change with time: the water level at the suction (or at delivery) might vary, modifying the static head. The temperature of the water might change with hot and cold season or the roughness of pipes might change with time, modifying the head losses.





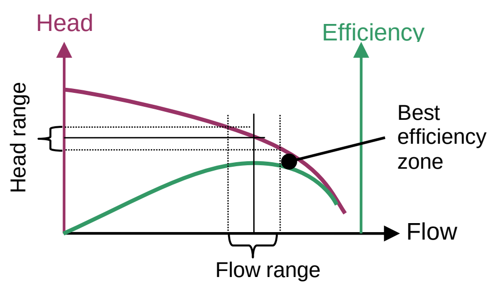
Flow range with variation of suction level



Flow range with variation of friction losses

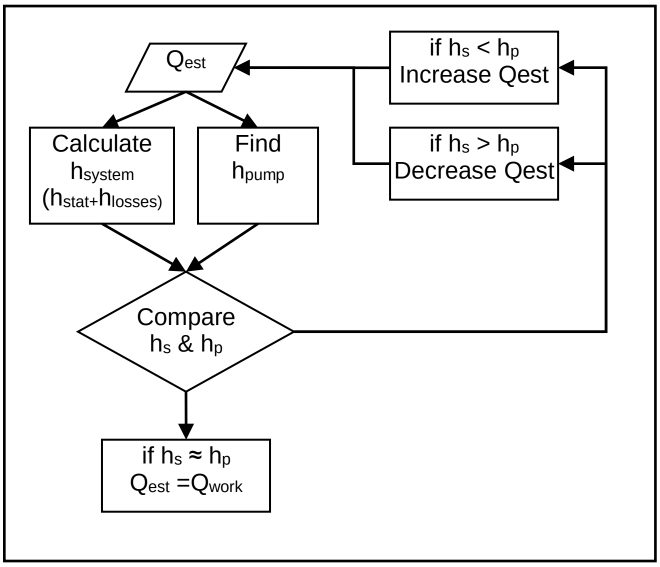
If these changes are significant, a working range should be defined for the pump. It should be checked that the pump is working safely throughout the whole range.

Duty point: once the head range (flow range) is found, the duty point can be selected in the middle of the range. This duty point should be as close as possible to the nominal flow of the pump where the efficiency is at its best.
If the flow range is too far from the nominal point, adjustment of the duty point should be done.



It is good to adapt the pumping working hours in order to have the duty point as close as possible to the optimal point of the pump.

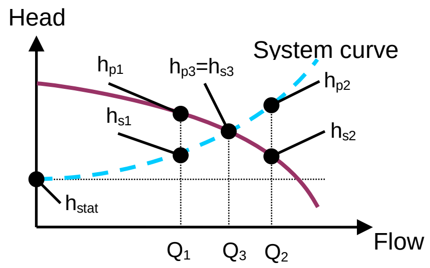
Necessary iteration to find the working point:
To calculate the head losses, the flow should be known, but the flow will be only known once the actual head losses are defined.
Therefore, a flow should be first estimated then the head of the system (static head plus losses) can be calculated and compare to the head of the pump. If head of the system is smaller, a bigger flow should be taken and vice versa, if the head of the system is bigger, a smaller flow should be taken.
Two or three iterations are usually enough to find a precise result.



Concretely, this can be done as follow:

1) Estimate a flow (Q_1), usually chosen as the nominal flow of the selected pump and find the corresponding head on the HQ pump characteristic (h_{pump})
2) Find the total head of the system for this flow, either by
   a. using graph in the annexes for head losses for pipe under pressure to estimate laminar friction losses and add punctual losses to them $$h_{LP} = h_L + k_P \cdot \frac{v^2}{2g}$$
   b. using formulas from chapter 4 to estimate λ, and therefore $$h_{LP} = \left(\lambda \cdot \frac{L}{D} + k_P\right)\frac{v^2}{2g}$$

   Then add the static head to the head losses to find h_{system}
3) Compare h_s & h_p
   a. If $h_s \approx h_p$ you have found the working flow, the process is completed.
   b. If $h_s > h_p$ you have you have to take a smaller flow
   c. If $h_s < h_p$ you have you have to take a bigger flow (in the example Q_2)
4) Place h_{stat} and h_{s1} on the pump characteristic to sketch the system curve as seen in § 4.9
5) Calculate the new total head for the system with Q_2.and plot it in the system curve. Find the intersection between the system and the pump characteristic, this should be the working flow (Q_3).
6) Check that the head of the system equal the head of the pump for Q_3, if it is not precise enough (3%) make additional iteration.



6.7. Adjustment to duty point

Pumps used for water supply systems are usually too small to be custom made. Therefore, to fit a given duty point, standard pumps must usually be adjusted. As represented in the next figures, standard pumps are made at a certain interval of flow and head covering a certain working range.

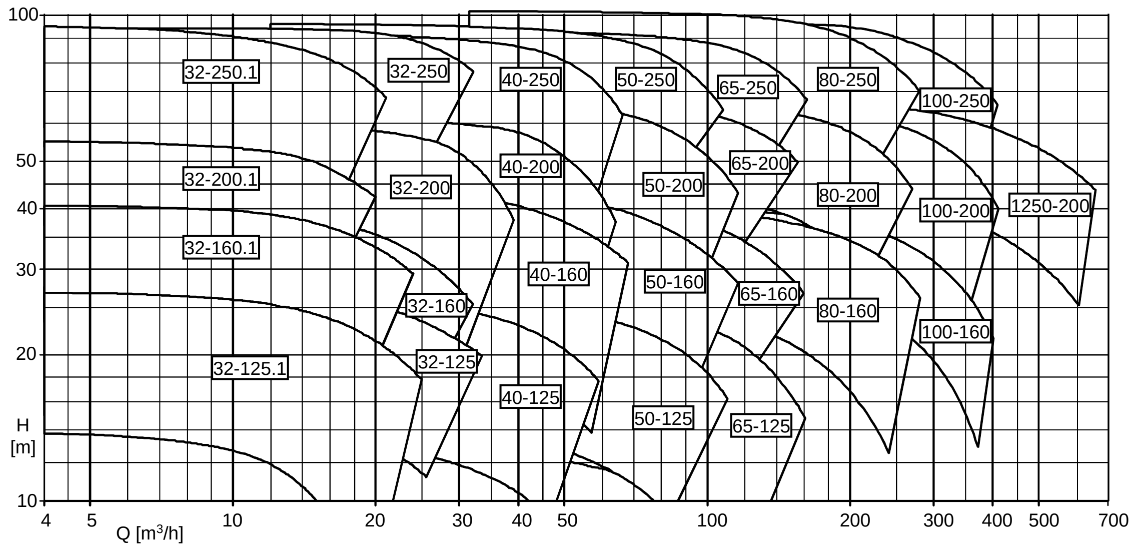
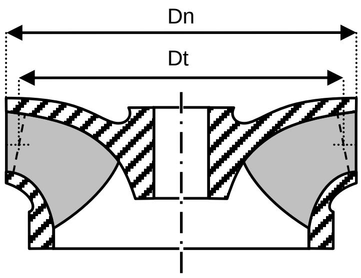


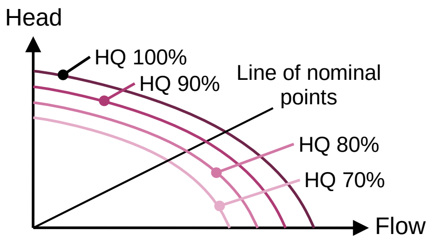
Fig 6- 3 Example of working range (KSB Etanorm pumps)

Trimming

To reduce permanently the head and the flow of an impeller, its external diameter (outlet of the blades) can be trimmed, reducing the outlet velocity of the fluid. In order not to affect performance, it can be done generally to a maximum of 80% of the initial diameter.

This method to adjust the duty point keeps a good efficiency, but it has to be done in the factory and is obviously irreversible and not possible with all types of impeller.





$$Q_t = Q_n \cdot \left(\frac{D_t}{D_n} \right)^2$$

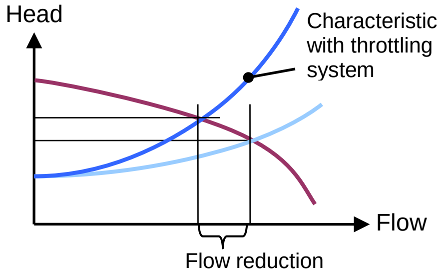
$$h_t = h_n \cdot \left(\frac{D_t}{D_n} \right)^2$$

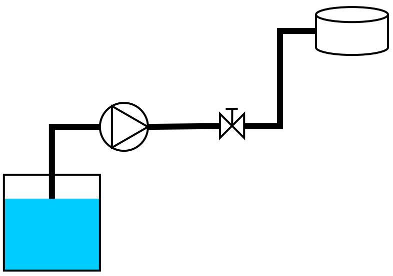
$$P_t = P_n \cdot \left(\frac{D_t}{D_n} \right)^4$$

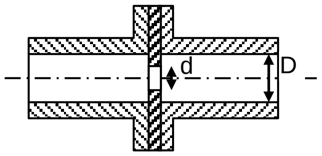
Eq. 6-7

Throttling

For **radial centrifugal pumps,** when the flow is slightly too high, it is possible to adjust it with a throttling system (orifice plate and or a globe valve). This will increase the losses on the system making its characteristics steeper.







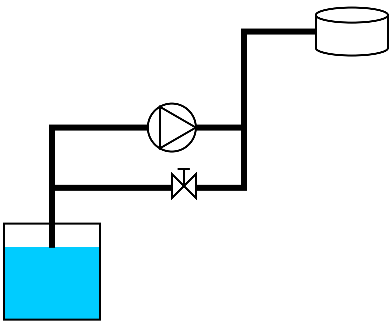
Installation of an orifice plate to be designed according to § 4.6

This is done for radial centrifugal pumps as it will decrease the needed power and save a bit of energy (cf §6.5).
In this case, the flow can even be smaller than the nominal flow of the pump.
This system has the big advantage of being adjustable and simple to install, well adapted when there is a big variation of the inlet level.

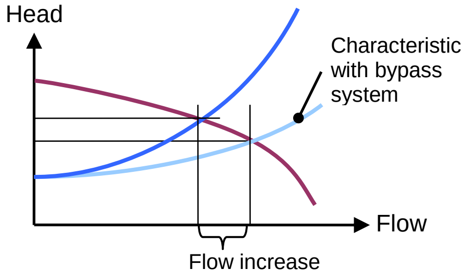
Bypassing

For **mixed or axial flow pump**, when the flow is slightly too small, it is possible to adjust it with a bypass system. This will make its characteristics less steep, increasing the flow.

This is done for mixed or axial flow pumps as it will decrease the needed power and save a bit of energy (cf §6.5).
In this case, the flow can even be higher than the nominal flow of the pump.
This system has the big advantage of being adjustable and simple to install, well adapted when there is a big variation of the inlet level.



This system should not be used for radial centrifugal pumps.



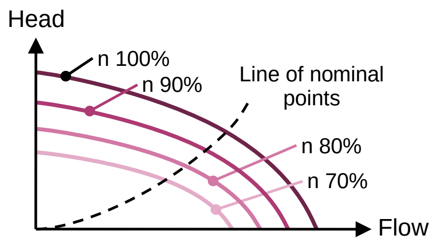
Speed reduction

By reducing the rotation speed of the pump, its flow is modified by the ratio of the speeds, its head by the square of this ratio and the power by the cube.
The nominal point is then changed accordingly and the efficiency for this new nominal point is almost not affected.
This is clearly the best way to regulate a pump but not the easiest to implement.

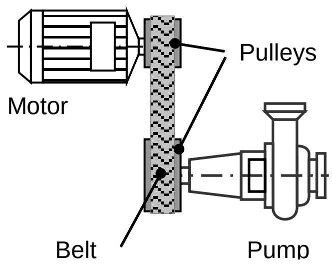
$$Q_2 = Q_1 \cdot {n_2}/{n_1}$$
$$h_2 = h_1 \cdot \left({n_2}/{n_1}\right)^2$$
$$P_2 = P_1 \cdot \left({n_2}/{n_1}\right)^3$$

Eq. 6-8

Q: flow [m^3/s]
h: head
P: power [W]
n_1: initial rotation speed
n_2: reduced rotation speed

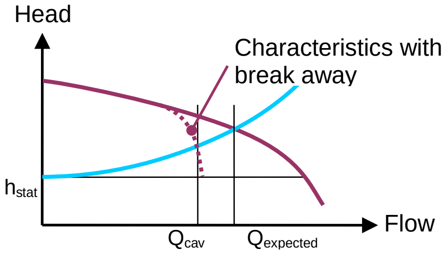


There are two ways to reduce the rotation speed:
The first way is by adding pulleys and a belt between the motor and the pump; it is rather simple, but not very good for bearings. It is less and less used.
The second one is to use an electronic device that will reduce the power frequency, thus the rotation speed according to Eq 6-5. These devices become cheaper but are still quite expensive for big power. As integrated electronic devices, they cannot be repaired and might be sensitive to low quality of power. Therefore, it is important to have a supplier locally available to change them when they are broken.

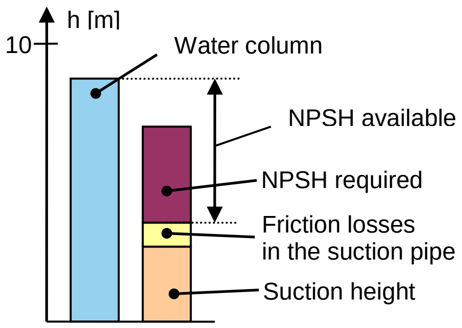


6.8. Cavitation in a pump and NPSH

The phenomenon of cavitation should absolutely be avoided in a pump; as seen in the chapter 2.3, if in any place in the pump the pressure drops below vapour pressure, steam bubbles will be created and when they will implode, it will destroy the impeller, the pump casing and or the delivery pipe. On top of this, the characteristic curve will be affected, falling suddenly, reducing the actual pumped flow; this phenomenon is known as breakaway as represented in the attached figure.



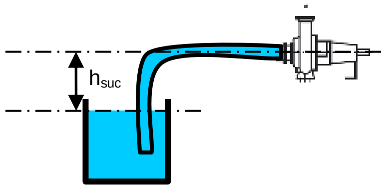
Three main factors will have effects on the decrease of the pressure in the pump: the suction height, the head losses in the suction pipe and the geometry of the pump (directly linked to the $NPSH_r$). The addition of this three "heights" should not exceed the water column height (as defined in §2.4), depending on the temperature and the altitude.
Thus for a given water system, the difference between the water column and the suction height plus the friction losses will give the maximum "height" available for the pump. This height is known as the **Net Positive Suction Head (NPSH) available**.



$NPSH_a$: NPSH available [m]
P_a: minimum atm pressure [Pa]
P_v: vapour pressure [Pa]
h_{LP}: hydraulic losses [m]
h_{suc}: suction height [m]

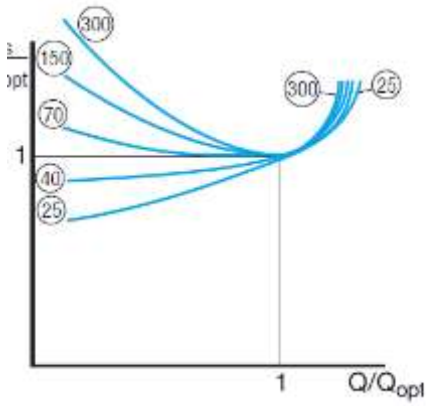
$$NPSH_a = \frac{P_a - P_V}{\rho \cdot g} - h_{LP} \pm h_{suc}$$

Eq. 6-9



The suction height is considered positive if the pump is placed below the water level.

The "height" needed by the pump is called **NPSH required**. As said, it depends on the geometry, as in a thinner channel the velocity of the water will be higher, thus the pressure lower, increasing the risk of cavitation. The rotation speed is therefore also very important, a small two pair of poles pump (n~2 900 rpm) will be more subject to cavitation than a bigger and more expensive four pair of poles pump (n~1 450 rpm).



As radial centrifugal pump have a good behaviour on cavitation at flow lower than their nominal points, axial flow pumps face cavitation due to formation of vortex in this situation as represented in the attached chart.

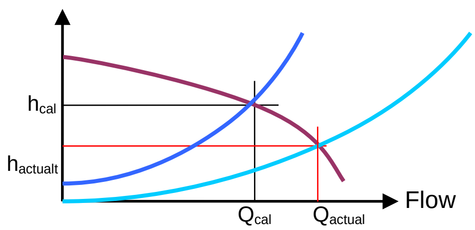
The calculation of the NPSH available should be done for the minimum and maximum suction water level, considering maximum losses in the suction pipe and minimum losses in the delivery pipe. If the NPSH available found is lower than the NPSH required by the pump, it should be considered to lower the elevation of the pump or to use an other type of pump.
Submersible pumps might also be subject to cavitation.

*6.9. Consequences of working over the maximum flow **

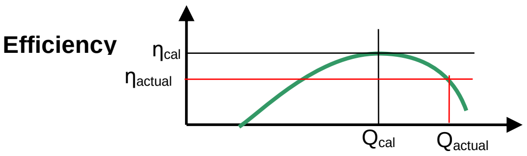
Most of the pumped used in the water supply system are radial centrifugal pumps (Ns<100). Therefore, it is essential not to over estimate losses and head as the following might happen.

By having, a calculated head higher than the actual head, the actual flow will be bigger.

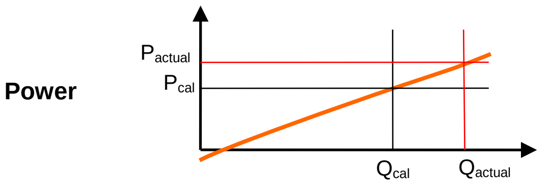
Head



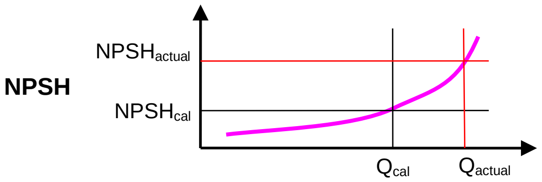
The calculated flow is usually at the optimal point, therefore with a bigger flow, the actual efficiency will be lower and energy will be wasted



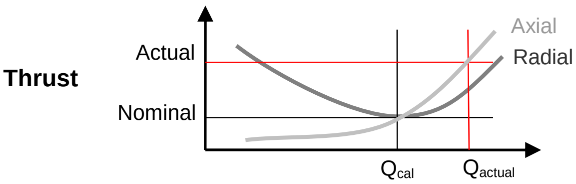
As the power will be higher than the rated one, if the nominal power of the motor is not big enough, there is a risk of overloading and over heating the motor and other electrical devices (genset, transformer, weirs, etc).



With the increased flow, the NPSH required will be higher, and if the suction head is too high, pump will have cavitation, wasting energy and damaging the impeller and pipes



Out of the optimal point, the axial and radial thrust are increased, this accelerate the wearing of the bearings and increase vibrations



With the increased velocity, the water hammer effect will be stronger and might affect the system, especially with steel pipes.

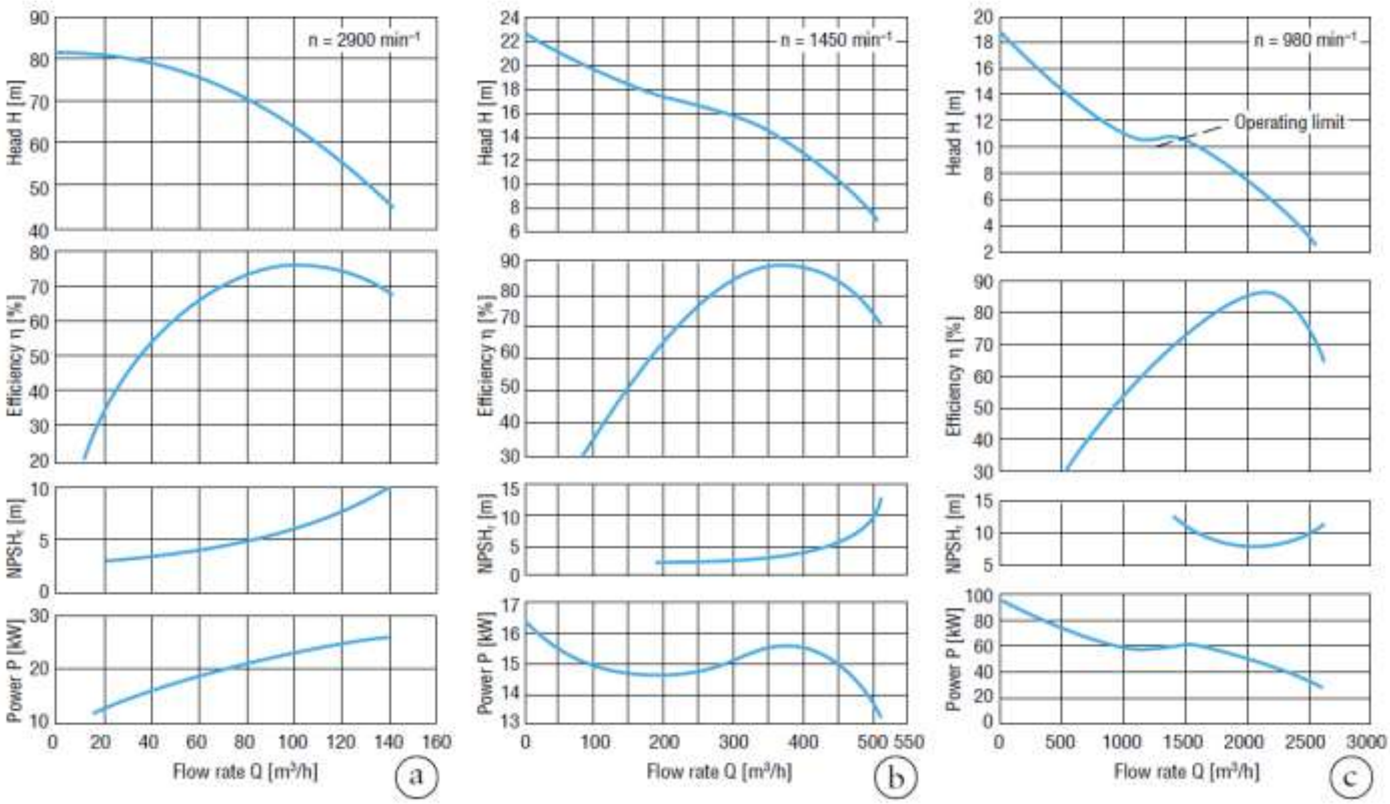
Therefore, to avoid these problems, a centrifugal pump has to be designed at its average working point and checked over the full range. When feasible, the desired flow should be adapted to existing pump nominal flow and the daily working hours adapted accordingly. There is no values that should be increased to be on the "safe side" but the suction height, which should be as small as possible to avoid cavitation.

References for this chapter

Grundofs : Pump Handbook :
KSB : Selecting Centrifugal Pumps

Basic exercises

1. For a pump of 50 m^3/h at 40 m head and with a 2 pair of poles motor ($N_S \approx 20$), what is the expected hydraulic power, pump efficiency, mechanical power, motor efficiency and power factor, active and total electrical power?
2. What are the nominal head, flow, efficiency, NPSH and power for the pumps a, b, and c?



3. What are the expected minimum and maximum flow for the pumps a, b, and c?
4. For the following system, what will be the duty point with the pump a, what is the power consumption?

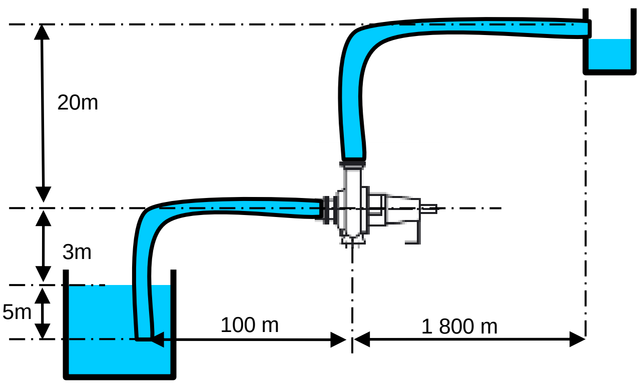
   Elevation : 200 masl

   Temperature 20°c

   All pipes are of new GI, DN 175

   Total punctual friction losses:

   k_p = 15 for suction part
   k_p = 5 for delivery part



Intermediary exercises

5. What is the number of pair of poles and the slip value of the motor for the pumps a, b, and c?
6. What is the specific speed for the pumps a, b, and c?
7. If we want to adjust the working point of the pump used in exercise 4 to the nominal point with a throttling system, what should be the size of the orifice, what would be the power consumption?
8. With the same situation as in exercise 4, what would be the new flow and power consumption, if we decrease the rotation speed, so that it reaches 80% of its initial value?
9. If the system is working as in exercise 4, what is the maximum suction height? And what is the maximum suction height if it is working as in exercise 7?

Advanced exercises

10. For the system used in exercise 7 (with a throttling system), knowing that the intake water level varies of plus or minus 2 meters, what would be the max and min flow, what would be the max suction height?
11. With a frequency inverter, what should be the percent speed reduction to adjust the system to the nominal point? What is the corresponding flow?

Chapter 7. Water hammer

What you will learn
Understand the phenomenon of water hammer, and parameters

Why
Water hammer can have important consequences on water system and need to be understood in order to avoid serious damage to the pipe network.

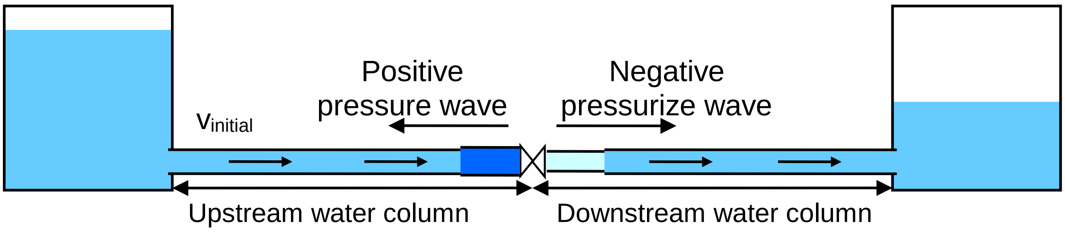
Duration of chapter 7
1 to 2 hours

*7.1. The principle of water hammer**

The theory seen in the chapter 3 and 4 as stated is only valid for permanent conditions where the fluid and the pipes are considered as incompressible. When flow conditions are changing slowly, Bernoulli's equations (Eq 3-2 to 3-4) can still be considered as valid and sufficient but when the conditions are suddenly changed, they cannot be applied any more. Typically, when a valve is suddenly closed at the end of a pipe, the flow is then stopped at its level but is still flowing at the entrance of the pipe. In this case, the continuity equation 3-2 is not valid any more (flow is still entering at one end but not going out anymore) and the flow cannot be considered as incompressible. Sudden changes in the flow are usually due to the following:

- opening and closure or modification of position of valves,
- starting, stopping or changing speed of pumps,
- changes in the flow due to operating valves (break pressure, flow controlling …),
- entrance or exit of air.

For a design, the analysis is done for the worst case scenario which correspond usually to the complete stop of the flow, subsequent to the closure of a valve or the sudden stop of pumps due to a power break (as in a normal operating procedures, valves are slowly closed before pumps are switch off if not conttolled by a soft starter).



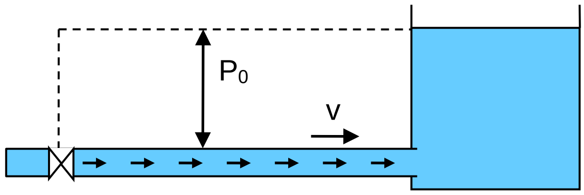
When a valve is suddenly closed in the middle of a pipe linking two tanks, as illustrated above, the inertia of the upstream water column will create an overpressure at the valve by compressing the water and expanding the pipe before the water is stopped. This change in the water condition (increase in pressure and decrease in velocity) will then be transmitted upstream segment by segment. Similarly, the inertia of the down steam water column leaving the valve, will create a depression of the water and a contraction of the pipe. This depression can easily reach the vapour pressure and then create cavitation in the pipe. These phenomena are well known as water hammer.

The situation in the pipe upstream and downstream is opposite but similar. The downstream situation will be studied, as it is what is happening in the delivery pipe of a pumping station, the case that concerns us the most.

The water hammer is a cyclical phenomenon. Each cycle can be divided into four phases as represented below.

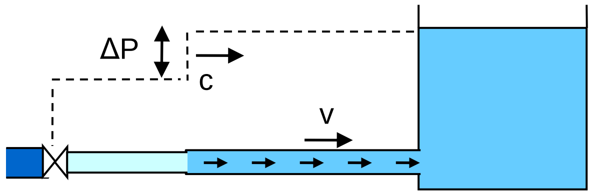
Before the valve is closed

A pressure P_0 exists in the pipe. In this example, we ignore friction losses, so the pressure is constant throughout the pipe.



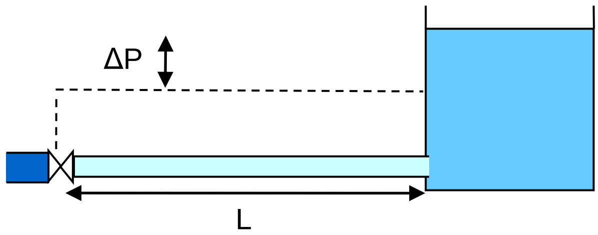
Phase 1 (t between 0 and L/c)

At T_0 the valve is closed. Due to inertia, a downstream wave begins to propagate along the pipe at a **speed c**. This wave will continue until it reaches the tank. In the depression zone, water is expanded and the pipe diameter is contracted, the speed of water is nil.



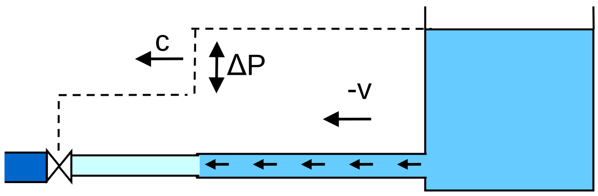
At t equals L/c

The pressure wave will reach the tank after a **time L/c**. Then, water has no more speed and is depressurized all along the pipe. As the water in the tank has a pressure higher than the one in the pipe, a reversed flow will start and thus, the wave will be reflected.



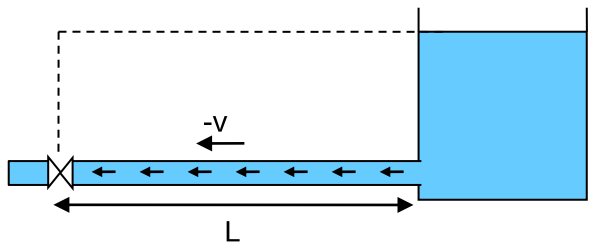
Phase 2 (t between L/c and 2L/c)

The pressure wave has been reflected by the tank and is being propagated always at a speed c, leaving behind a pressure equals to the initial pressure. As the pipe volume increases and water is recompressed; thus, to fill in this volume water has to flow from the tank into the pipe. Therefore, a flow is generated backwards from the initial flow at a same but opposite speed v.



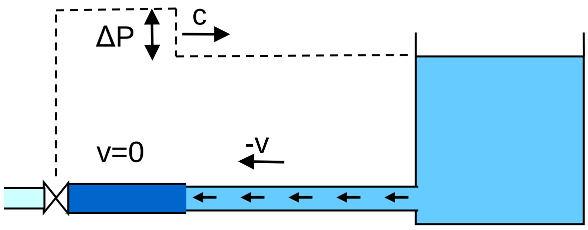
At t equals 2L/c

The pressure wave has reached the valve and the situation in the downstream pipe is now similar as the one in the upstream pipe before the valve closure. The pressure equals the initial pressure in the whole pipe and the speed of water is -v in the whole pipe. Thus, the wave will be reflected as an overpressure.



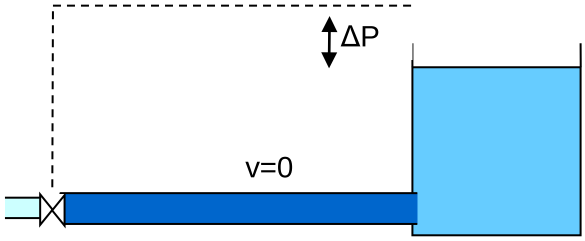
Phase 3(t between 2L/c and 3L/c)

The overpressure is being propagated at a speed c. In the pressured zone, water is compressed and the pipe diameter is expanded, the speed of water is nil.



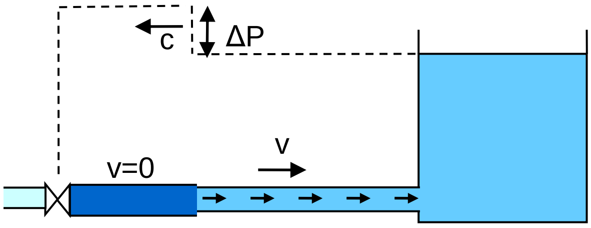
At t equals 3L/c

The pressure wave reaches again the tank after a time of 3L/c. Then, water has no more speed and is pressurized all along the pipe. As the water in the tank has a pressure lower than the one in the pipe, a reversed flow will start and thus, the wave will be reflected.



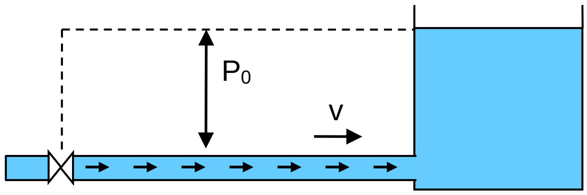
Phase 4 (t between 3L/c and 4L/c)

The pressure wave leaves behind a flow at initial pressure. As the pipe is being contracted again and water decompressed, water must flow out of the pipe (towards the tank). Therefore, water will flow in its original direction and at its initial velocity.



At t equals 4L/c

The wave reaches the valve and the whole pipe is under initial pressure and velocity. The pressure wave will be reflected and the whole cycle will begin again with an underpressure wave travelling downstream.



Thus, we can see that conditions (pressure and flow) are changing over time and along the pipe. The following charts illustrate the pressure situation over time for different point along the pipe. Those charts will help us to see if the over pressure exceed the maximum PN allowed for the pipe and if the underpressure might go below vapour pressure, breaking the water column with a cavitation pocket.

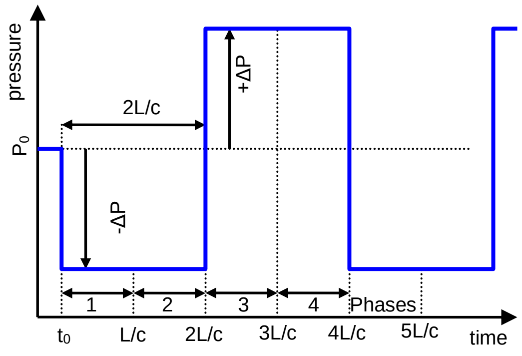
This cyclic phenomenon is mainly attenuated by the hydraulic losses, but can last for a long while if they are small.

In order to avoid that the over pressure will blow-up the pipe or that the depressure will collapse a plastic pipe, gasket or just suck dirt into the pipe through the leakages, it is important to be able to estimate the pressure through the time along the pipe.

In the following charts, head losses absorption and closure time are ignored. It will be shown afterwards which effect they have and how to take them in to consideration.

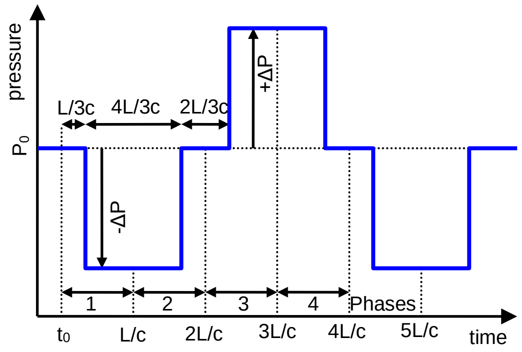
At the valve

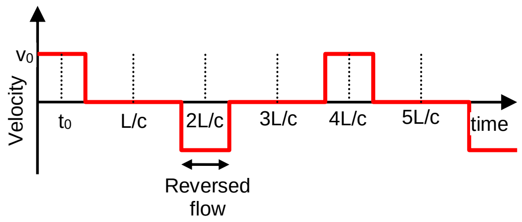
Assuming an instantaneous closure of the valve at the time t_0, the pressure is instantaneously decreased by the inertia of the water column. Once the pressure wave comes back after having been reflected by the tank (after t=2L/c), the flow is reversed in the pipe. Thus, the valve has to "stop" the flow and is subject to an overpressure due to the inertia of the water column. The cycle will then start again after the pressure waves come back for the second time (t=4L/c).



At a third of the pipe

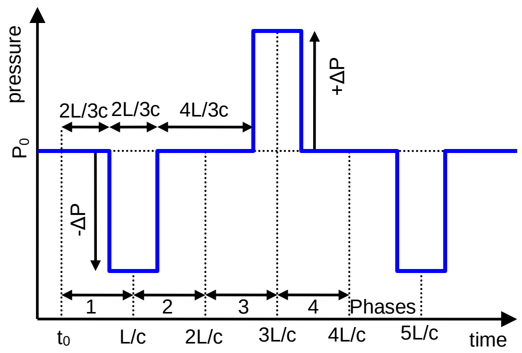
The pressure wave takes t=L/3c to reach this point, during this time the flow and pressure keep their initial values. Then the water will be stopped and the pressure will drop. The condition will be constant until the pressure wave comes back from the tank after t=4L/3c. It will then regain its initial pressure and have its flow reversed but of the same velocity as the initial flow. The pressure wave will then be reflected by the valve and come back after t=2L/3c stopping the flow. Therefore, the pressure will rise until the wave comes back from the tank where it will re-establish the initial conditions.

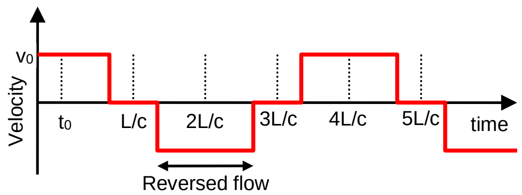




At two third of the pipe

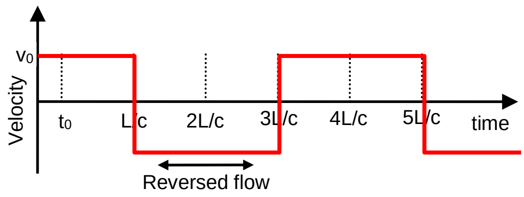
If the situation is monitored at a point situated at two third of the pipe (between the valve and the tank), the phenomenon will be similar but the pressure peak periods shorter and the time with water velocity longer. Thus the first wave will come after two third of L/C, have the wave back after the same time and then have a reverse flow for two time two third of L/C, for the wave to reach the valve and come back.





In the tank

The pressure is constant, although the level will increase and decrease according to its diameter but this can be neglected. The velocity of the water at the entrance of the tank will be constant till the surge arrive after L/c, then it will be reversed for two time L/c before it find its original value.



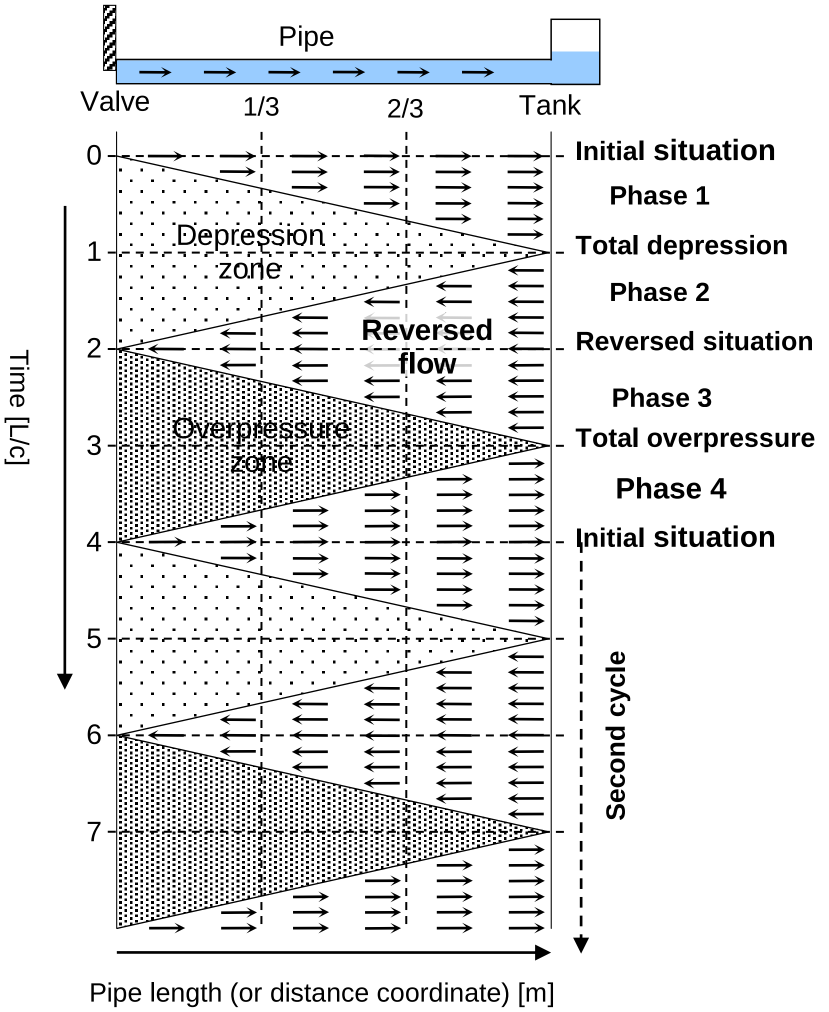
An useful way to represent what is happening in the pipe through the time is shown in the attached chart.

In horizontal, the pipe length is given and in vertical, it shows the evolution through the time.

The arrows are showing the flow direction in the pipe.

The light spotted areas are the zone of the pipe with depression and heavy spotted area with over pressure.

By following the dotted line at 1/3 or 2/3 of the pipe length, the pressure and flow velocity of the previous charts can be figured out.



7.2. *Formula of water hammer**

Joukowsky has shown in 1898 that the pressure surge in the water hammer is directly proportional to the initial water speed, the velocity of the pressure wave and the density of the water. This formula is only if there is no cavitation and not too much gas (air) in the water.
It should be noticed that the pressure surge is not dependent of the length of the pipe.

- ΔP: pressure surge [Pa]
- ρ: density of water [kg/m^3]
- Δv: variation of water velocity [m/s]
- c: velocity of pressure wave [m/s]
- Δh: head surge [m]
- g: earth gravity

$$\Delta P = \rho \cdot \Delta v \cdot c$$
$$\Delta h = \Delta v \cdot \frac{c}{g} \quad \textbf{Eq. 7-1}$$

This equation is also valid for variation of water speed. It means that if the velocity will change of 10% the pressure surge will be of 10% of what it would be in the worst case.

The velocity of pressure wave depends on the Young's modulus of the pipe, the bulk modulus of water, the wall thickness and diameter of the pipe:

c: velocity of the pressure wave
K_{water}: bulk modulus of water (2.2 GPa)
K_{pipe}: Young's modulus of pipe (see section 1.3)
D: internal diameter of the pipe
e: wall thickness of the pipe
ρ: density of water (1000 kg/m^3)

$$c = \sqrt{\frac{K_{water}/\rho}{1 + \frac{D}{e} \cdot \frac{K_{water}}{K_{pipe}}}}$$ **Eq. 7-2**

Velocity of the pressure wave is close to the sound speed in the water (1450 at 10 °C) in "hard" pipes and can decrease quickly according to materials and thickness. The following table shows range of expected values for typical materials.

Type of pipe	Bulk modulus K [GPa]	Velocity of the pressure wave c [m/s]	Pressure surge for Δv=1 m/s ΔP [bar]
Steel	160 - 200	Up to 1 485 m/s	up to 15 bar
Cast iron	80 - 170	900 to 1 300 m/s	9 to 13 bar
Concrete	30 - 50		
PVC	3.0 - 4.7	320 to 680 m/s	3 to 6 bar
PE	0.7 – 1.2	200 to 400 m/s	2 to 4 bar

The time taken by the wave to go to the tank and come back is given by the following equation:

T_r: Return time of the wave
L: length of the pipe
c: speed of pressure wave

$$T_r = \frac{2 \cdot L}{c}$$ **Eq. 7-3**

The return time is an important parameter of water system, allowing setting closure time for valve or shutdown time for pump soft starters avoiding water hammer as explained in section 7.4.

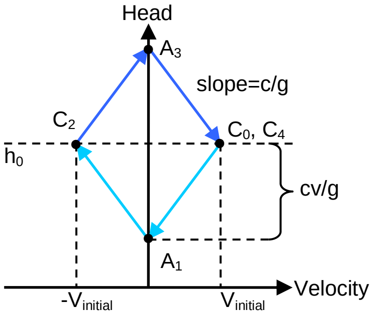
More values that are detailed are given in the Annexe N

7.3. Graphical methodology of Schnyder-Bergeron

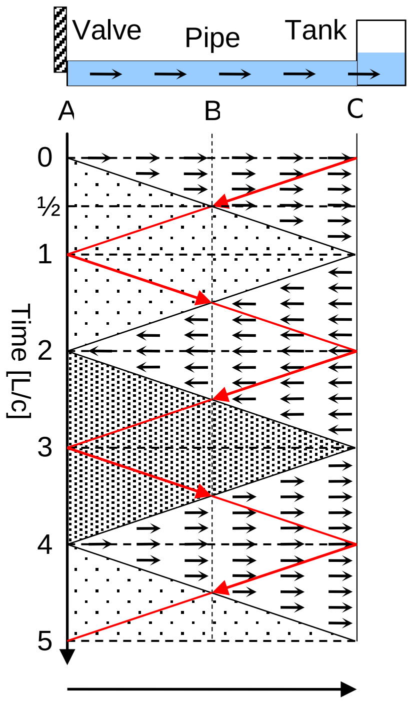
Based on simplification of the water hammer equations, the Schnyder-Bergeron or characteristics methodology allows a rough graphical estimation of pressure surges and facilitates the understanding of the phenomenon in different situations. It can be used in quite complicate system and it was the methodology most used before the computer age.
Its principle is to follow an observer travelling through the system at the velocity of the pressure wave, as indicated by the red lines in the time / pipe length attached chart. If the observer crosses a pressure surge, its new condition (velocity / head) will be found by at the intersection of the characteristics (line with a slope c/g) and the known parameter.

For instance for a pipe connected to a tank with a given head (h_0) facing a sudden closure of a valve with an initial velocity (v_0) and with losses neglected.
In this basic case our observer will start from the tank at t=0 (C_0), meet the surge at t=L/c/2 ($B_{½}$) and reach the valve at t=L/c (A_1). Then he will go back, crossing the surge at $B_{1½}$ and reaching the tank at C_2 and so on as represented by the red line.

In the characteristics graph, the point C_0 represent the initial condition, this position will be kept till the surge is met then the new position will be the intersection of the characteristic line with the known velocity (V=0) thus allowing to find the depressure value (cv/g) for the position A_1. This condition will be kept until the wave back is met ($B_{1½}$), then the new position (C_2) will be the intersection of the characteristic line but with a negative slope and the known head (the tank level as losses are neglected). The same is done to find A3 and C4 bring the observe back to the initial position.



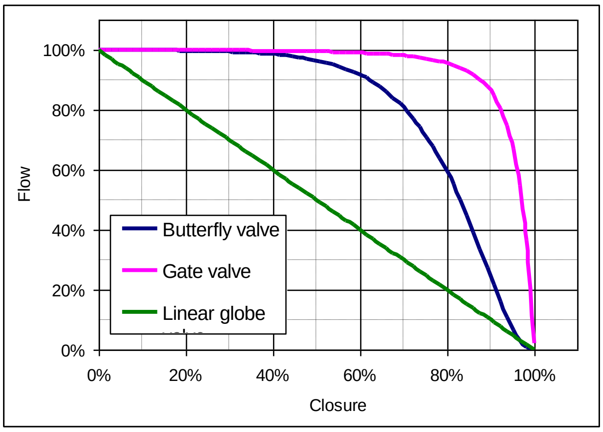
Characteristics graph for a sudden closure without head losses



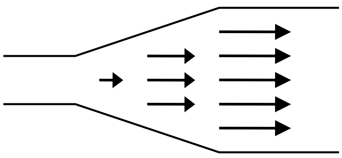
Those values can then be used to draw the charts of head and velocity / time at the A, B, C or any desired positions.
In more complex situation, additional "observers" starting at a different time are added to know other intermediary conditions.

7.4. *Effect of the closure time**

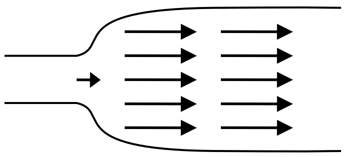
The closure time (T_c) is the time during which the flow varies. For example, in the case of a power cut, the pump will not stop instantaneously but will gradually reduce its flow depending on its inertia and water pressure.
Similarly, valves need some time to be closed. The way the flow is reduced is usually not linear, for instance, gate valves have their flow reduced mainly during the last 20% of the closure run. The attached chart is showing the actual flow reduction according to the closure run for a gate valve, a butterfly valve and a linear globe valve.



Thus, the wave front of the pressure surge will depend on the closure characteristic and speed. Without this information the over and under pressure along the pipe cannot the properly defined as it will be illustrated in the section about envelopes for pumping stations.



Front wave of linear globe valve closure



Front wave of gate valve closure

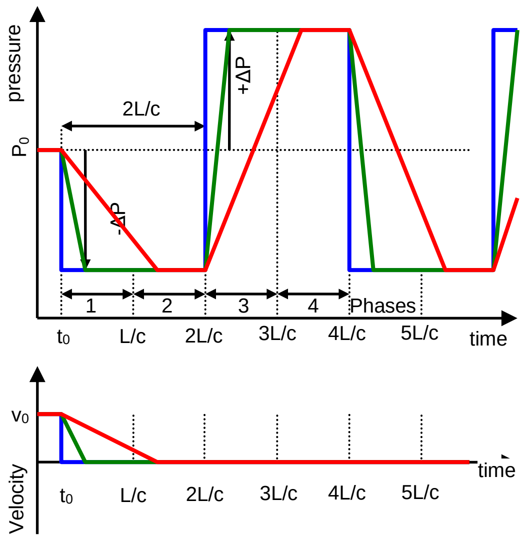
The charts below show the pressure and velocity over time at the valve and in the middle of the pipe for a instantaneous closure ($T_c = 0$) in blue, a quick closure ($T_c = T_r/6$)in green and a slower closure ($T_c = 4T_r/6$) in red.

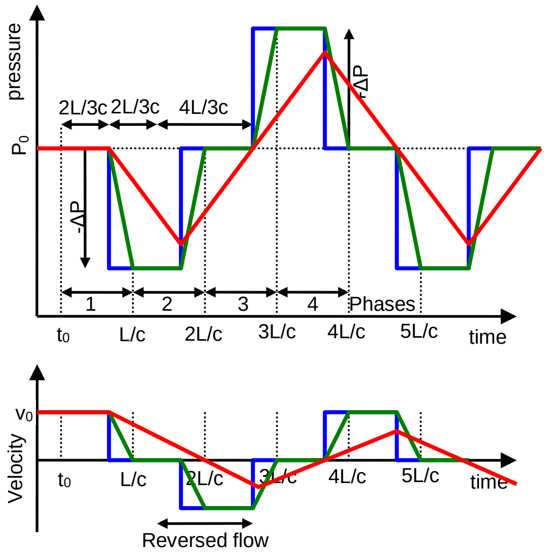
At the valve

The pressure surge is attenuated in the quick and slower closure case but the maximum pressure is not changed. The velocity, once the valve is closed, remains zero.

At the middle of the pipe

As at the valve level, the quick closure has not much effect on the pressure surge, but with the slower closure the pressure surge is reduced.

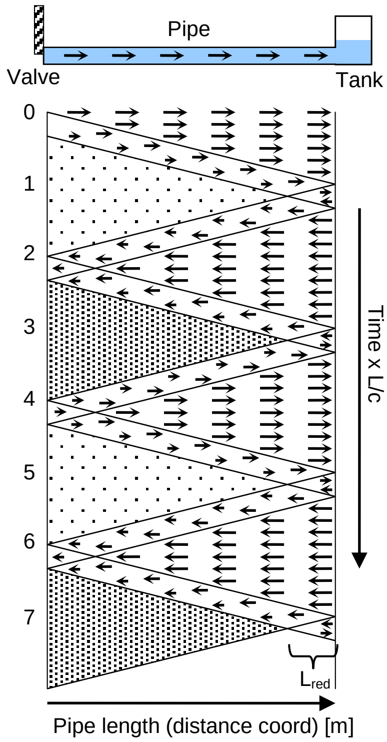




When the wave front is reflected by the tank, there is a reduction of the pressure surge by the superposition of the surge, slower is the closure time, longer will be the part of the pipe with a reduction of the pressure surge.

If the closure time is greater than the return time of the valve, then the water hammer effect will be also diminished at the valve level and equation 7-1 has to be used with Δv instead of the initial velocity. On top of that, the attenuation of the water hammer will be faster as additional friction losses are added to the system.

Simulation of a pump stopping is similar as a valve closure, however its stopping time will depend mainly of its inertia and head. For submersible or horizontal multistage pumps, the inertia is quite small compared to the head and the closure should be estimated as instantaneous. For bigger pump like double volute type, the stopping time can be quite important thus this information should be requested from the supplier.



Following equation gives the length of the pipe, starting form the tank, with reduced pressure surge.

L_{red}: length of the pipe with reduced surge
T_c: Closure or stopping time
c: velocity of the pressure surge

$$\boxed{L_{red} = T_c \cdot c/2}$$ **Eq. 7-4**

If the closure time is bigger than the return time, then the all pipe will have a reduce head surge. For simple situation with a linear front wave, the following equation gives roughly the maximum head surge at the valve during the slow closure.

Δh_{max}: max head surge [m]
L: total length of the pipe [m]
v_{in}: initial velocity of the water m/s]
g: earth gravity
T_c: Closure or stopping time

$$\text{If } T_c > T_r \qquad \Delta h_{max} = \frac{2L \cdot v_{in}}{g \cdot T_c}$$ **Eq. 7-5**

In this case, the head surge is not depending any more on the velocity of the pressure wave, thus the material of the pipe. Therefore, for system with a possibility to have a slow closure, the head surge might not depend on the material used.

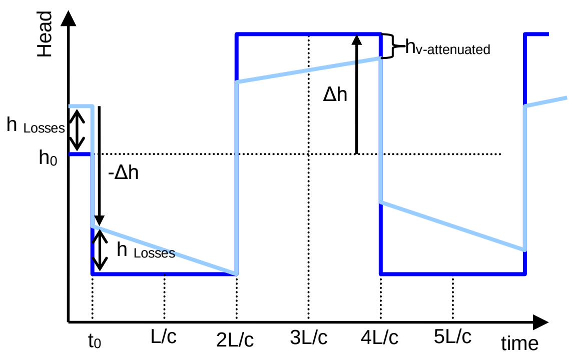
For calculation more precise, Schnyder Bergeron or computer simulation should be used especially if the closure time is very long, if the friction losses are important or if the front wave is not linear.

7.5. Effect of head losses

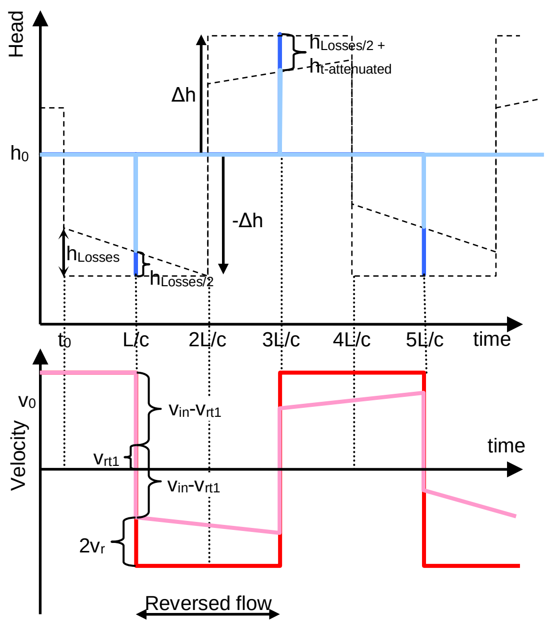
Influences of head losses in the water hammer phenomena are complex and difficult to calculate, but their consequences might be quite important and not very intuitive. In this section, the effect of the phenomena will be roughly explained and a simplified way to calculate the attenuated pressure surge will be explain.

In the following explanation, it is assumed that the valve is instantaneously closed so that the front wave of the surge is strait.

At the valve

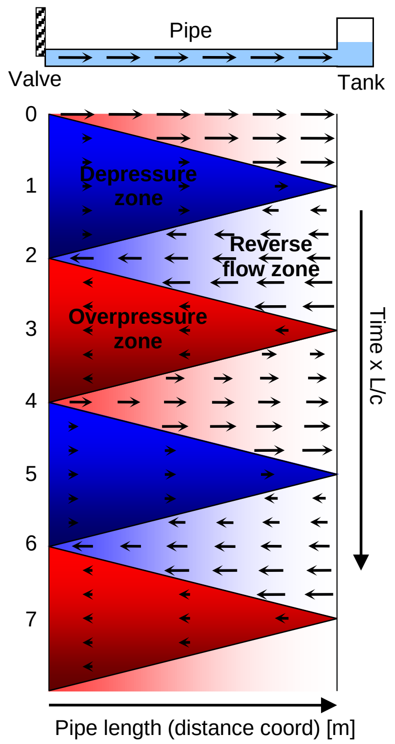


Just before the tank



These illustrations show the difference of a system where the head losses were neglected (in dark blue/red) and a system where they were taken into consideration (in light blue/pink).

At the initial time (t_0) at the valve, the difference between the heads values (illustrated by the dark blue and light blue lines) are the losses. Right after the closure, both will drop of the same value -Δh. In the case with losses neglected, the pressure will stay constant until the return of the surge and will be constant along the pipe, from the valve until the tank.



In the second case, the pressure will be constant throughout the zone under depressure but will decrease slowly over time (as illustrated by the blue colour becoming darker in the pipe distance chart) till it reaches the same depressure as with losses neglected just before the return time (2L/c). This reduction of pressure is due to the fact that the head surge, getting closer to the tank, will be more important, inducing in the pipe a small flow that will keep a the pressure constant throughout the depressure zone at a given time. This flow is constant at a given place during the depressure time but is increasing along the pipe, passing from zero at the valve to its maximum (v_{rt1}) just before the tank. This phenomenon is not at intuitive and clearly not respecting the Bernoulli's law specifying that the flow along a pipe is constant. Thus, when the surge is reflected at the tank, as the velocity is not nil there, the reflection velocity will be reduced of twice this remaining velocity (as illustrated in the velocity / time chart.
It is important to notice that the first negative surge was not attenuated by the head losses.
In the reversed flow zone, the flow is this time following the Bernoulli's law, as it is the same through out the pipe, increasing slowly over the time, until the surge arrives for the second time from the valve. Thus, a new gradient of pressure is created in the pipe due to the headlosses (as illustrated with the white to light blue colour in the reverse flow zone).
In the overpressure zone, the situation will be similar as the one described for the under pressure, but as the maximum velocity of the reverse flow is smaller than the initial velocity, the maximum over pressure (at t=4L/C) will be these time smaller then in the system where head losses were neglected.

Thus we can see that the first depressure is not attenuated at the valve but is gradually attenuated along the pipe till a value of $h_{losse}/2$ at the tank. Then the over pressure is much more attenuated from the valve till the tank. The graphical methodology gives us rather good approximations for the attenuations of the first circle of depressure (illustrated in the previous page chart), if the head losses are not too important, slightly over estimating them.

$$h_{v\text{-}att} = \frac{\left(\sqrt{1+2\frac{k_{LP} v_{in}}{c}} - 1\right)^2 c^2}{2 g k_{LP}} \quad \textbf{Eq. 7-6}$$

$h_{v\text{-}att}$: attenuation of head surge at the valve [m]
c: velocity of pressure wave [m/s]
k_{LP}: friction coefficient [-] as per eq 4-10
v_{in}: initial velocity [m/s]
g: earth gravity [m/s^2]

v_{rt1}: remaining velocity at the tank [m/s]
$h_{t\text{-}att}$: attenuation of head surge at the tank [m]

$$v_{rt1} = \frac{k_{LP} v_{in}^2}{4c}$$

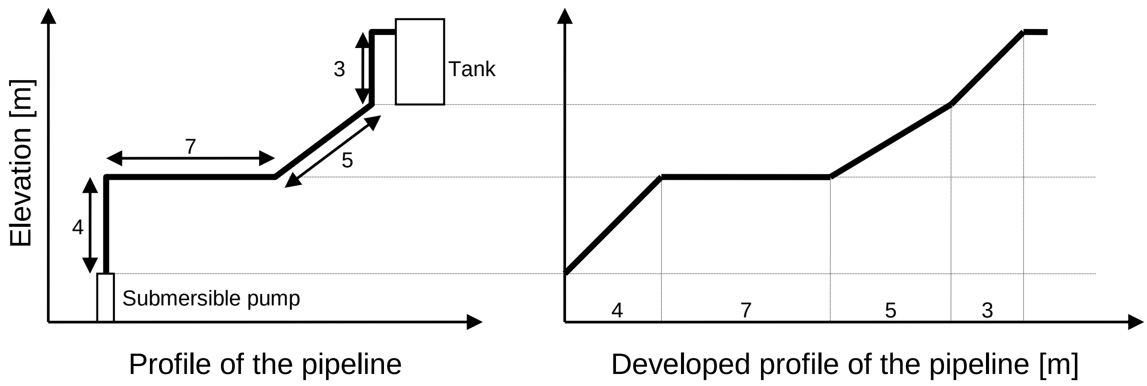
$$h_{t\text{-}att} = \frac{2\left(\sqrt{1+\frac{k_{LP}\left(v_{in} - v_{t1}\right)}{c}} - 1\right)^2 c^2}{g k_{LP}} \quad \textbf{Eq. 7-7}$$

An illustration of these attenuations is given in the next section.

*7.6. Water hammer envelopes for pipelines**

The aim of the envelop schemes is to represent the value of the over pressure and the depressure along the pipeline and to see if they do not exceed acceptable values. If it is the case, the design of the pipeline should be revised or a water hammer protection system has to be added. The envelopes are usually found by computer, but for simple or simplified systems, it can be done manually. Even if the result is not very accurate, it helps to understand the main issues about the planed system and to adapt the design if needed.

The first step is to draw a scheme with the developed profile of the pipe, versus the elevation. For profile with small slopes (< 25%) there is no big differences and it can be neglected, but for system with bigger slopes, it has to be taken in consideration as shown below. For system with vertical pipes (like boreholes their representation would be a 45° angle.



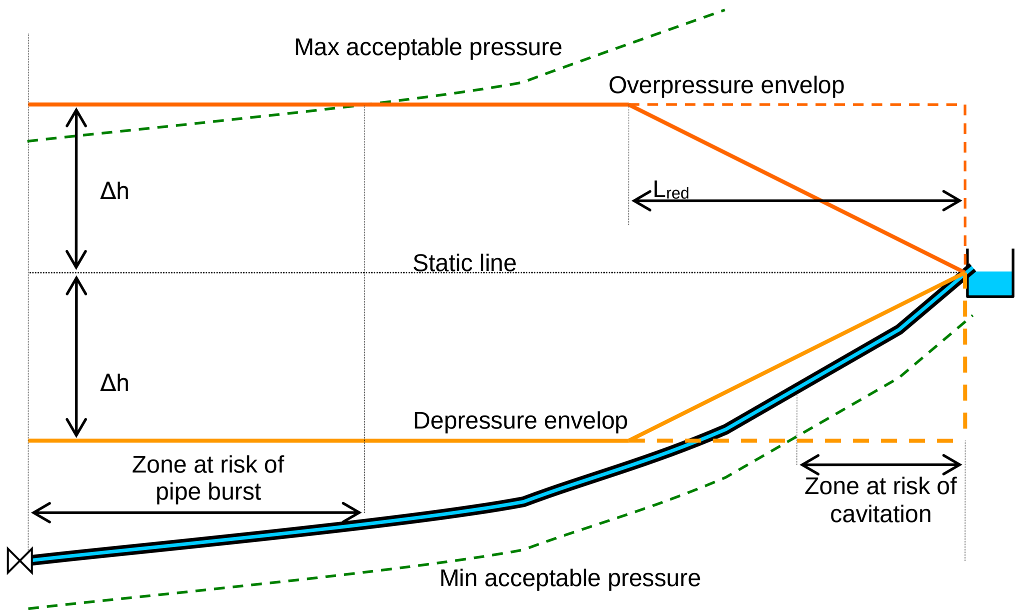
Comparaison of a profile and a developed profile for a system with a submersible pump.

The second step is to plot the acceptable values for pressure (in dashed green in the scheme):

- The maximum acceptable pressure line will be draw according to the rated nominal pressure (PN) above the pipeline. As the units are meter, the distance should be at PN/ρg, or approximated PN [bar]·10.
- The minimum acceptable pressure would be the vapour pressure where water will become steam, breaking by cavitation the water column. This can have serious effect when the surge comes back, the over pressure will be significantly increased. This height (as per equation eq. 2-4), as seen in chapter 2.4 depends of the temperature and elevation but can be approximated to 8m. In general, for drinking water supply, it is not recommended to have this minimum pressure below the pipeline level, (not having pressure lower than the atmospheric pressure) so that dirty water might not be sucked in the system in the leakage or gaskets between pipes pulled inside the pipes. Plastic pipes are also at risk of collapse in this case.

The last step is to draw the envelope of the water hammer, if the head losses are neglected:

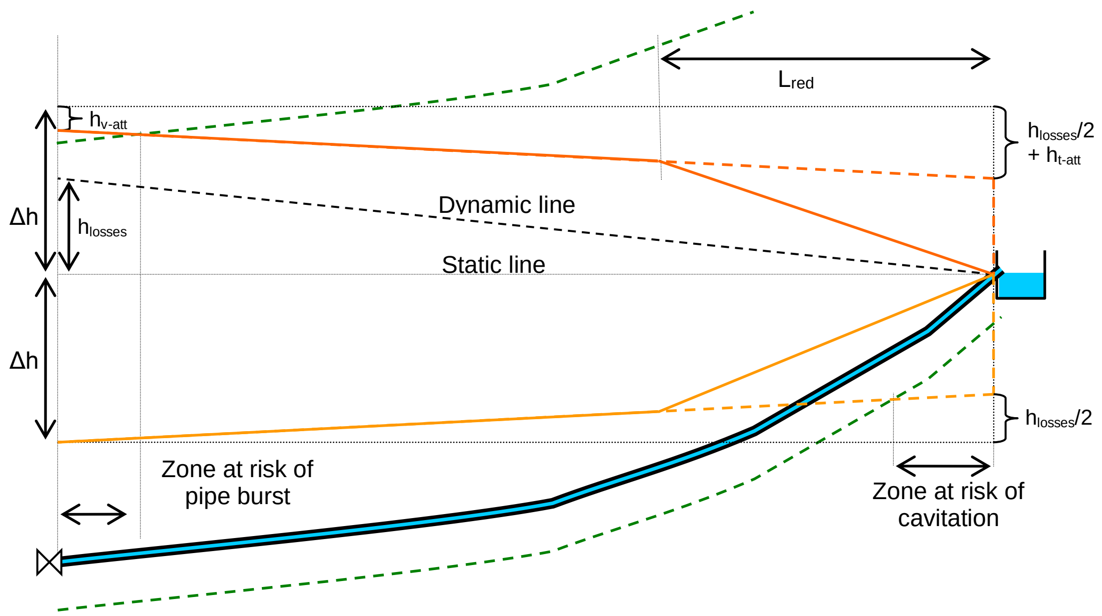
- Draw the static line of the system (horizontal line at the tank level).
- Calculate Δh according to eq 7.1
- Draw the lower level of the envelope, horizontal line at Δh below the static line.
- Draw the upper level of the envelope, horizontal line at Δh above the static line.
- Take into consideration the closure or stopping time by making a "biseau" starting from the tank and of a length according to eq.7.4 , representing the section of the pipe with reduced pressure surge, as show below.



Envelope without head losses

If head losses are taken into consideration:

- Draw the static line of the system (horizontal line at the tank level).
- Calculate h_{losses} according to eq. 4.10.
- Draw the dynamic line of the system (dashed line starting at h_{stat} + h_{losses} and reaching the tank)
- Calculate Δh according to eq 7.1, $h_{v\text{-}att}$ and $h_{t\text{-}att}$ according to eq 7.6 & 7.7.
- Draw the lower level of the envelope, line starting at Δh below the static line and reaching the tank at Δh - h_{losses}/2.
- Draw the upper level of the envelope, line starting at Δh-$h_{v\text{-}att}$ above the static line and reaching the tank at Δh - h_{losses}/2 - $h_{t\text{-}att}$.
- Take into consideration the closure or stopping time by making a "biseau" starting from the tank and of a length according to eq 7.4.



Envelope with head losses

The actual minimum pressure in the pipeline is represented by the difference of elevation between the pipe level and the lower envelope, and the maximum pressure, by the difference between the pipe level and the upper envelope.
If the envelope is between the two dashed green lines (max and min acceptable pressure) all along the pipeline, it should resist to the pressure surge. If part of it is above the max acceptable pressure line, it will be at risk to burst, and it part of it is below the min acceptable line, it will be at risk of cavitation or collapse as illustrated in the schemes.
In this case, the design of the pipeline should be review or a water hammer protection system has to be added.
It can be seen that the effect of the closure (stopping) time is very significant to protect the end part of the pipeline against cavitation. Thus, a particular attention should be taken to find this actual value from the pump supplier. A soft starter, allowing controlling the stopping time of the pump might help significantly, but will not act in case of power break.
It can be noted that if the losses are neglected, the zone at risk is much bigger than when it is taken into consideration, thus overestimating the needs for protection.

Review pipeline design
The following possibilities will have effect on the water hammer surge. If their applications are not too expensive they have the advantage to avoid any additional system that might fail or brake and will anyway need maintenance, thus jeopardising the sustainability of the water system.

- To **increase the PN** of the pipes, will allow higher acceptable pressure in the first part of the pipe line.
- **Change the profile** of the pipeline, having a concave profile will help to avoid cavitation at the end section or high point section of the pipeline.
- **Change the material**, use PVC or PE instead of metallic pipe will reduce significantly the head surge.
- **Increase the diameter** of the pipe, will reduce the velocity of the water, thus proportionally the water surge.

Water hammer protection system

- A **pump by-pass** system, to attenuate the depressure at the beginning of the pipe
- Add a **flywheel** at the pump to increase the stopping time
- Add **air valve** to protect high point at the end of the pipeline.
- Anti water hammer **vessel** for system with a limited pipeline length
- Water **chimney** to release the flow, adapted for convex profile
- Pressure release valve to avoid high pressure in part of the pipe.

The solution to avoid water hammer should also be adapted to its probability or frequency. to optimise its price.

This simplified methodology helps to understand the main point to have a good design for the water system, but can not substitute a computer simulation for more complex situations such as system with multiple pumps, with branches, with different diameters, etc.

References for this chapter:

KSB: water hammer
Advanced water distribution

Basic exercises

1. Draw the pressure and the velocity versus the time (L/c) at the middle of the pipe for an instantaneous valve closure, neglecting head losses.

Using the tables in the annexe N

2. What are the velocity (c), the head surge (Δh) and the return time of the pressure wave in a cast iron pipe DN200 of 3 kilometres for a instantaneous decrease of velocity of 1.5 m/s?
3. What are the velocity (c), the head surge (Δh) and the return time of the pressure wave in a PE pipe SDR11 of 5 kilometres for a instantaneous decrease of velocity of 2 m/s?
4. What are the velocity of the pressure wave (c), the head surge (Δh) and the return time in a PVC pipe SDR17, OD 200 of 2 kilometres for a instantaneous closure with an initial velocity of 1 m/s?
5. What length of the previous pipe will have a reduced surge if the decreasing time is of 4.1 second?
6. Neglecting losses draw the envelope scheme for the previous pipe, knowing that its profile is as per the attached table. Is the pipe safe (assuming a linear front wave)? If not, what can be done?

	Pump	Pt 1	Pt 2	Pt 3	Tank
Pipe length	0	500	1'000	1'500	2'000
Elevation	0	10	30	30	70

7. For the previous system what should be the closure time to limit the Δh at 30m?

Intermediary exercises

8. Draw the pressure and the velocity versus the time (L/c) at the middle of the pipe for an instantaneous valve closure, taking into consideration head losses.

Using equation 7-1, 7-2 & 7-3

9. What are the velocity (c) the head surge (Δh) and the return time (T_r) of the pressure wave in a cast iron pipe DN200 (e=6.4 mm, K_{CI}=120 GPa) of 3 km, if a pump of 170 m^3/h instantaneously stops working, with a temperature of water of 45°?
10. For the previous pipe, we want to limit the head surge at 100m by having a longer stopping time, what should be in this case T_C?
11. What are the velocity (c) the head surge (Δh) and the return time (T_r) of the pressure wave in a PE80 pipe PN12.5, OD200 (K_{PE}=0.7 GPa, e=18.2mm) of 5 km, if a pump of 150 m^3/h instantaneously stops working, with a temperature of water of 20°?
12. What are the friction coefficient (K_{LP}) and the head losses (h_{LP}) of the previous pipe with a roughness of 0.07 mm, neglecting the punctual losses? What is the attenuation of the negative head surge due to the losses at the tank?
13. What are the attenuation of head surge at the valve ($h_{v\text{-}att}$), the remaining velocity at the tank (v_{t1}) and the attenuation of the positive head surge at the tank ($h_{t\text{-}att}$) of the previous pipe?

14. For the previous pipe, draw the developed profile of the pipeline, the static and dynamic lines, and the envelope schemes knowing that its profile is as per the attached table. Is the pipe safe? If not, what can be done?

	Pump	Pt 1	Pt 2	Pt 3	Tank
Pipe length	0	1'000	1'000	1'000	2'000
Elevation	0	0	5	10	30

15. What would be the effect on the head surge on the previous exercises of having a Young's modulus (K_{PE})of 1.1 GPa instead of 0.7 GPa?

Advanced exercises

16. For exercise 1-4, what would be the change in OD (DN for metallic pipe) following the change in pressure due to water hammer?
17. Draw the Schnyder Bergeron diagram for the water hammer with head losses

List of annexes

Annexe A. Conversion of units

Length 1		centimetre	inch	foot	yard	meter	rod
centimetre	cm	1	2.54	30.48	91.4	100	502.9
inch	in	0.3937	1	12	36	39.37	198
foot	ft	0.0328	0.083	1	3	3.28	17
yard	yd	0.0109	0.0278	0.333	1	1.09	6
meter	m	0.0100	0.0254	0.305	0.914	1	5.03
rod	rd	0.0020	0.0051	0.061	0.182	0.199	1

Length 2		chain	arpent	kilometre	mile (land)	mile (naut)	league
chain	ch	1	2.91	49.7	80	92.1	240
arpent	arp	0.3440	1	17.1	27.5	31.7	82.6
kilometre	km	0.02012	0.05847	1	1.609	1.852	4.828
mile (land)	mi	0.0125	0.0363	0.621	1	1.15	3
mile (naut)	nmi	0.0109	0.0316	0.540	0.869	1	2.61
league	lea	0.00417	0.0121	0.207	0.333	0.384	1

Area		sq cm	sq inch	sq foot	sq yard	sq meter	sq rod
sq cm	cm^2	1	6.452	929.0	8 361	10'000	252 929
sq inch	sq in	0.155	1	144	1296	1550	39 204
sq foot	sq ft	0.00108	0.00694	1	9	10.76	272
sq yard	sq yd	1.20×10^{-4}	7.72×10^{-4}	0.111	1	1.20	30.3
sq meter	m^2	10^{-4}	6.45×10^{-4}	0.0929	0.836	1	25.3
sq rod	sq rd	3.95×10^{-6}	2.55×10^{-5}	0.00367	0.0331	0.0395	1

Volume		litre	gallon (UK)	gallon (US)	cu foot	barrel (UK)	cu meter
liter	l	1	3.785	4.546	28.32	163.7	1 000
gallon (UK)	gal	0.264	1	1	7	43.23	264
gallon (US)	gal	0.220	0.833	1	6	36.00	220
cu foot	cu ft	0.0353	0.134	0.161	1	5.78	35
barrel (UK)	bl	0.00611	0.0231	0.028	0.173	1	6.11
cu meter	m^3	0.001	0.00379	0.005	0.0283	0.164	1

Flow		litre p. sec	cub. meter p. hour	gallon p. min. UK	gallon p. min. US	litre per minute	cub. meter per day
litre p. sec	l/s	1	3.6	13.20	15.85	60	86.4
cub. meter p. hour	m^3/h	0.28	1	3.67	4.40	16.67	24
gallon p. min. UK	GPM	0.0758	0.273	1	1.20	4.55	6.55
gallon p. min. US	GPM	0.0631	0.227	0.833	1	3.79	5.45
litre per minute	l/min	0.0167	0.06	0.22	0.264	1	1.44
cub. meter per day	m^3/d	0.0116	0.0417	0.153	0.183	0.694	1

Mass		gram	ounce	pound	kilogram	hundredweight short	hundredweight long
gram	g	1	28.350	453.592	1000	45 359	50 802
ounce	oz	0.0353	1	16	35.3	1600	1792
pound	lb	0.00220	0.0625	1	2	100	112
kilogram	kg	0.001	0.0283	0.454	1	45.36	50.8
hundredweight short	cwt sh	2.20×10^{-5}	6.25×10^{-4}	0.01	0.0220	1	1.12
hundredweight long	cwt lg	1.97×10^{-5}	5.58×10^{-4}	0.00893	0.0197	0.893	1

Speed		meter p. sec	knot	miles p. hour	foot p. sec	km p. hour	inch p. sec
meter p. sec	m/s	1	0.5144	0.4470	0.3048	0.2778	0.0254
knot	kt	1.94	1	0.869	0.592	0.540	0.0494
miles p. hour	mph	2.24	1.15	1	0.682	0.621	0.0568
foot p. sec	fps	3.28	1.69	1.47	1	0.911	0.0833
km p. hour	km/h	3.60	1.85	1.61	1.10	1	0.0914
inch p. sec	ips	39.4	20.3	17.6	12.0	10.9	1

Energy		joule	calorie	BTU	horsepower -hour	kilowatt-hour	inch-pound force
joule	J	1	4.187	1 055	2.685×10^{6}	3.6×10^{6}	0.1130
calorie	cal/s	0.239	1	252	6.41×10^{5}	8.60×10^{5}	0.0270
BTU	BTU	9.48×10^{-4}	0.00397	1	2'546	3'414	1.07×10^{-4}
horsepower -hour	hp	3.73×10^{-7}	1.56×10^{-6}	3.93×10^{-4}	1	1.34	4.21×10^{-8}
kilowatt-hour	hp	2.78×10^{-7}	1.16×10^{-6}	2.93×10^{-4}	0.746	1	3.14×10^{-8}
inch-pound force		8.85	37.1	9'334	2.38×10^{7}	3.19×10^{7}	1

Power		watt	calorie/s	BTU p. min	horsepower metric	horsepower UK
watt	W	1	4.187	453.6	135.5	746
calorie/s	cal/s	0.2388	1	108	32.4	178
BTU p. min	BTU/m	0.00220	0.00923	1	0.299	1.64
horsepower metric	hp	0.00738	0.0309	3.35	1	5.51
horsepower UK	hp	0.00134	0.00561	0.608	0.182	1

Temperature		Definition	Conversion to kelvin
degree Celsius	°C	°C ≡ K − 273.15	[K] ≡ [°C] + 273.15
degree Delisle	°De		[K] = 373.15 − [°De] × 2/3
degree Fahrenheit	°F	°F ≡ °C × 9/5 + 32	[K] ≡ ([°F] + 459.67) × 5/9
degree Newton	°N		[K] = [°N] × 100/33 + 273.15
degree Rankine	°R;	°R ≡ K × 9/5	[K] ≡ [°R] × 5/9
degree Réaumur	°Ré		[K] = [°Ré] × 5/4 + 273.15
degree Rømer	°Rø		[K] = ([°Rø] − 7.5) × 40/21 + 273.15
kelvin (SI base unit)	K		≡ 1 K

Annexe B. Theoretical diameter for PE pipes

			SDR 6			SDR 7.4			SDR 9			SDR 11			SDR 13.6			SDR 17			SDR 21			SDR 26		
			S 2.5			S 3.2			S 3.9811			S 5.0119			S 6.3096			S 8			S 10			S 12.5		
	PE 63		-			-			-			PN 10			PN 8			-			PN 5			PN 4		
	PE 80		PN 25			PN 20			PN 16			PN 12.5			PN 10			PN 8			PN 6			PN 5		
	PE 100		-			PN 25			PN 20			PN 16			PN 12.5			PN 10			PN 8			PN 6		
OD	Max	Ovality	e_{min}	e_{max}	ID	e_{min}	e_{max}	ID	e_{min}	e_{max}	ID	e_{min}	e_{max}	ID	e_{min}	e_{max}	ID	e_{min}	e_{max}	ID	e_{min}	e_{max}	ID	e_{min}	e_{max}	ID
16	16.3	1.2	3.0	3.4	10.0	2.3	2.7	11.4	2.0	2.3	12.0															
20	20.3	1.2	3.4	3.9	13.2	3.0	3.4	14.0	2.3	2.7	15.4	2.0	2.3	16.0												
25	25.3	1.2	4.2	4.8	16.6	3.5	4.0	18.0	3.0	3.4	19.0	2.3	2.7	20.4	2.0	2.3	21.0									
32	32.3	1.3	5.4	6.1	21.2	4.4	5.0	23.2	3.6	4.1	24.8	3.0	3.4	26.0	2.4	2.8	27.2	2.0	2.3	28.0						
40	40.4	1.4	6.7	7.5	26.6	5.5	6.2	29.0	4.5	5.1	31.0	3.7	4.2	32.6	3.0	3.5	34.0	2.4	2.8	35.2	2.0	2.3	36.0			
50	50.4	1.4	8.3	9.3	33.4	6.9	7.7	36.2	5.6	6.3	38.8	4.6	5.2	40.8	3.7	4.2	42.6	3.0	3.4	44.0	2.4	2.8	45.2	2.0	2.3	46.0
63	63.4	1.5	10.5	11.7	42.0	8.6	9.6	45.8	7.1	8.0	48.8	5.8	6.5	51.4	4.7	5.3	53.6	3.8	4.3	55.4	3.0	3.4	57.0	2.5	2.9	58.0
75	75.5	1.6	12.5	13.9	50.0	10.3	11.5	54.4	8.4	9.4	58.2	6.8	7.6	61.4	5.6	6.3	63.8	4.5	5.1	66.0	3.6	4.1	67.8	2.9	3.3	69.2
90	90.6	1.8	15.0	16.7	60.0	12.3	13.7	65.4	10.1	11.3	69.8	8.2	9.2	73.6	6.7	7.5	76.6	5.4	6.1	79.2	4.3	4.9	81.4	3.5	4.0	83.0
110	110.7	2.2	18.3	20.3	73.4	15.1	16.8	79.8	12.3	13.7	85.4	10.0	11.1	90.0	8.1	9.1	93.8	6.6	7.4	96.8	5.3	6.0	99.4	4.2	4.8	101.6
125	125.8	2.5	20.8	23.0	83.4	17.1	19.0	90.8	14.0	15.6	97.0	11.4	12.7	102.2	9.2	10.3	106.6	7.4	8.3	110.2	6.0	6.7	113.0	4.8	5.4	115.4
140	140.9	2.8	23.3	25.8	93.4	19.2	21.3	101.6	15.7	17.4	108.6	12.7	14.1	114.6	10.3	11.5	119.4	8.3	9.3	123.4	6.7	7.5	126.6	5.4	6.1	129.2
160	161.0	3.2	26.6	29.4	106.8	21.9	24.2	116.2	17.9	19.8	124.2	14.6	16.2	130.8	11.8	13.1	136.4	9.5	10.6	141.0	7.7	8.6	144.6	6.2	7.0	147.6
180	181.1	3.6	29.9	33.0	120.2	24.6	27.2	130.8	20.1	22.3	139.8	16.4	18.2	147.2	13.3	14.8	153.4	10.7	11.9	158.6	8.6	9.6	162.8	6.9	7.7	166.2
200	201.2	4.0	33.2	36.7	133.6	27.4	30.3	145.2	22.4	24.8	155.2	18.2	20.2	163.6	14.7	16.3	170.6	11.9	13.2	176.2	9.6	10.7	180.8	7.7	8.6	184.6
225	226.4	4.5	37.4	41.3	150.2	30.8	34.0	163.4	25.2	27.9	174.6	20.5	22.7	184.0	16.6	18.4	191.8	13.4	14.9	198.2	10.8	12.0	203.4	8.6	9.6	207.8
250	251.5	5.0	41.5	45.8	167.0	34.2	37.8	181.6	27.9	30.8	194.2	22.7	25.1	204.6	18.4	20.4	213.2	14.8	16.4	220.4	11.9	13.2	226.2	9.6	10.7	230.8
280	281.7	9.8	46.5	51.3	187.0	38.3	42.3	203.4	31.3	34.6	217.4	25.4	28.1	229.2	20.6	22.8	238.8	16.6	18.4	246.8	13.4	14.9	253.2	10.7	11.9	258.6
315	316.9	11.1	52.3	57.7	210.4	43.1	47.6	228.8	35.2	38.9	244.6	28.6	31.6	257.8	23.2	25.7	268.6	18.7	20.7	277.6	15.0	16.6	285.0	12.1	13.5	290.8
355	357.2	12.5	59.0	65.0	237.0	48.5	53.5	258.0	39.7	43.8	275.6	32.2	35.6	290.6	26.1	28.9	302.8	21.1	23.4	312.8	16.9	18.7	321.2	13.6	15.1	327.8
400	402.4	14.0				54.7	60.3	290.6	44.7	49.3	310.6	36.3	40.1	327.4	29.5	32.5	341.0	23.7	26.2	352.6	19.1	21.2	361.8	15.3	17.0	369.4
450	452.7	15.6				61.5	67.8	327.0	50.3	55.5	349.4	40.9	45.1	368.2	33.1	36.6	383.8	26.7	29.5	396.6	21.5	23.8	407.0	17.2	19.1	415.6
500	503.0	17.5							55.8	61.5	388.4	45.4	50.1	409.2	36.8	40.6	426.4	29.7	32.8	440.6	23.9	26.4	452.2	19.1	21.2	461.8
560	563.4	19.6										50.8	56.0	458.4	41.2	45.5	477.6	33.2	36.7	493.6	26.7	29.5	506.6	21.4	23.7	517.2
630	633.8	22.1										57.2	63.1	515.6	46.3	51.1	537.4	37.4	41.3	555.2	30.0	33.1	570.0	24.1	26.7	581.8
710	716.4	24.9													52.2	57.6	605.6	42.1	46.5	625.8	33.9	37.4	642.2	27.2	30.1	655.6
800	807.2	28.0													58.8	64.8	682.4	47.4	52.3	705.2	38.1	42.1	723.8	30.6	33.8	738.8
900	908.1	31.5																53.3	58.8	793.4	42.9	47.3	814.2	34.4	38.3	831.2
1000	1009	35.0																59.3	65.4	881.4	47.7	52.6	904.6	38.2	42.2	923.6
1200	1209	42.0																			57.2	63.1	1085.6	45.9	50.6	1108.2
1400	1409	49.0																						53.5	59.0	1293.0
1600	1609	56.0																						61.2	67.5	1477.6

Annexe C. Diameter for PVC pipes

C=2.5

S=(OD-e)/2e SDR=OD/e	Tolérance	S20 SDR 41 PN 5		S16.7 SDR34.4 PN6		S16 SDR33 PN6.3		S12.5 SDR26 PN8		S10 SDR21 PN10		S8 SDR17 PN12.5		S6.3 SDR13.6 PN16		S4 SDR9 PN25	
OD		e	ID	e	ID	e	ID	e	ID	e	ID	e	ID	e	ID	e	ID
10	0.3															1.5	7.0
12	0.3															1.5	9.0
16	0.3													1.5	13.0	1.8	12.4
20	0.3													1.5	17.0	2.3	15.4
25	0.3											1.5	22.0	1.9	21.2	2.8	19.4
32	0.3									1.6	28.8	1.9	28.2	2.4	27.2	3.6	24.8
40	0.3					1.5	37.0	1.6	36.8	1.9	36.2	2.4	35.2	3	34.0	4.5	31.0
50	0.3					1.6	46.8	2	46.0	2.4	45.2	3	44.0	3.7	42.6	5.6	38.8
63	0.3	1.6	59.8	1.9	59.2	2	59.0	2.5	58.0	3	57.0	3.8	55.4	4.7	53.6	7.1	48.8
75	0.3	1.9	71.2	2.2	70.6	2.3	70.4	2.9	69.2	3.6	67.8	4.5	66.0	5.6	63.8	8.4	58.2
90	0.3	2.2	85.6	2.7	84.6	2.8	84.4	3.5	83.0	4.3	81.4	5.4	79.2	6.7	76.6	10.1	69.8

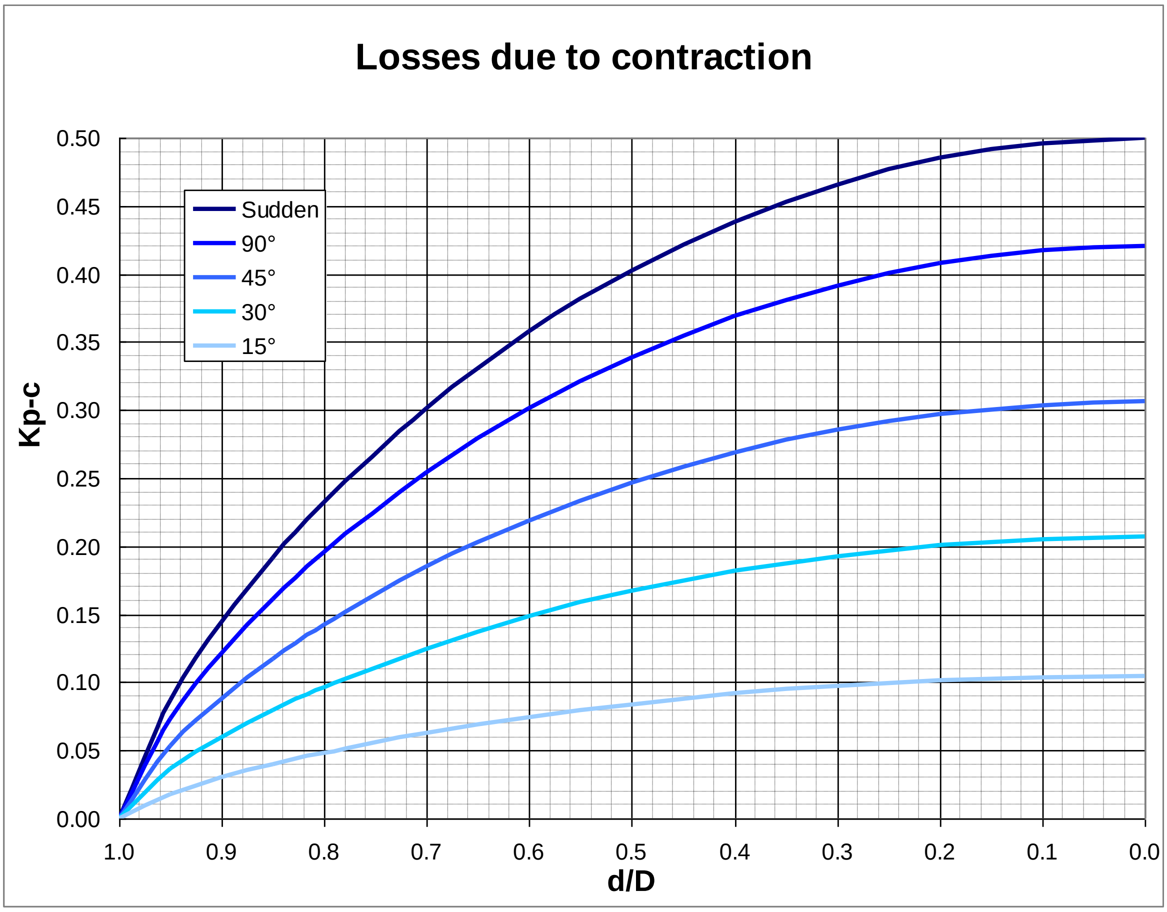
C=2

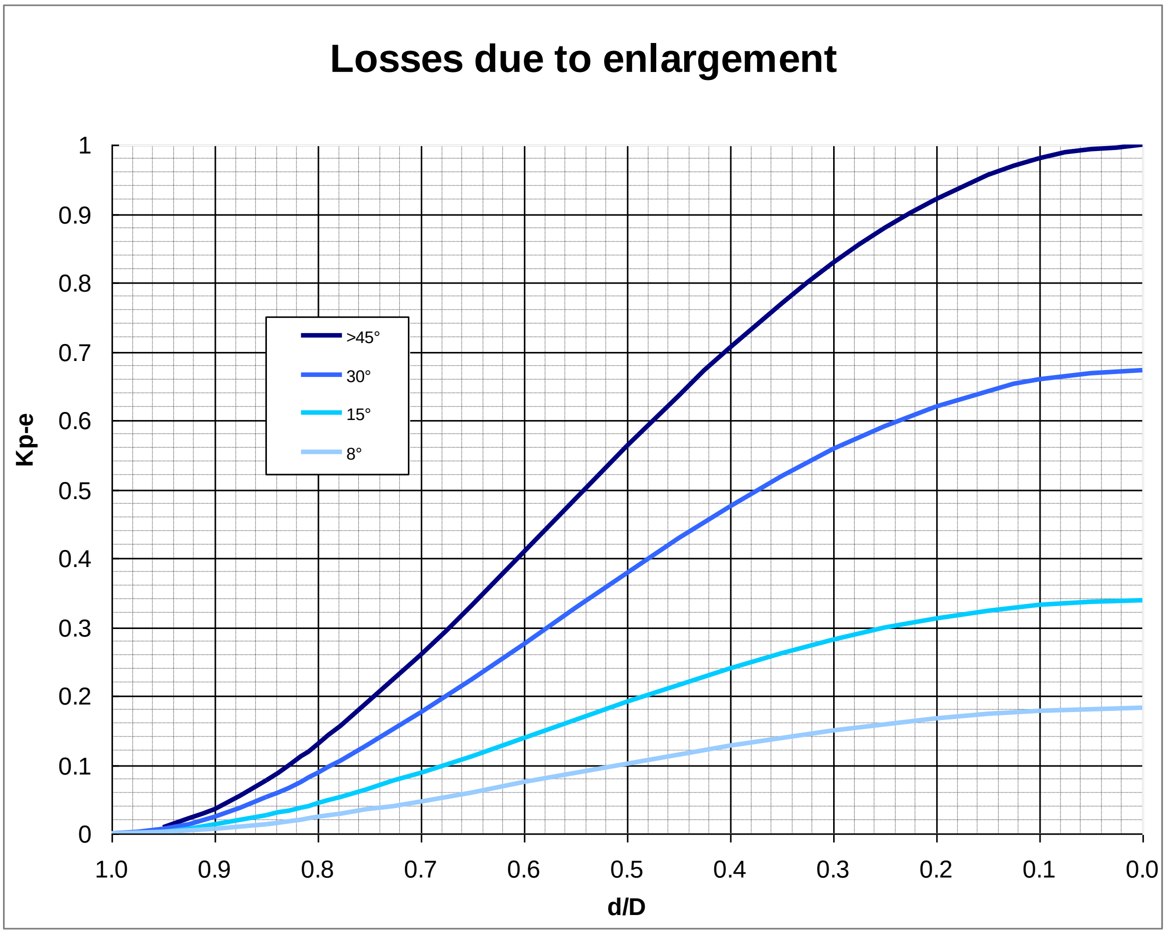
S=(OD-e)/2e SDR=OD/e		S20 SDR41 PN6.3		S16 SDR33 PN8		S12.5 SDR26 PN10		S10 SDR21 PN12.5		S8 SDR17 PN16		S6.3 SDR13.6 PN20		S5 SDR11 PN25	
OD		e	ID	e	ID	e	ID	e	ID	e	ID	e	ID	e	ID
110	0.4	2.7	104.6	3.4	103.2	4.2	101.6	5.3	99.4	6.6	96.8	8.1	93.8	10.0	90.0
125	0.4	3.1	118.8	3.9	117.2	4.8	115.4	6.0	113.0	7.4	110.2	9.2	106.6	11.4	102.2
140	0.5	3.5	133.0	4.3	131.4	5.4	129.2	6.7	126.6	8.3	123.4	10.3	119.4	12.7	114.6
160	0.5	4.0	152.0	4.9	150.2	6.2	147.6	7.7	144.6	9.5	141.0	11.8	136.4	14.6	130.8
180	0.6	4.4	171.2	5.5	169.0	6.9	166.2	8.6	162.8	10.7	158.6	13.3	153.4	16.4	147.2
200	0.6	4.9	190.2	6.2	187.6	7.7	184.6	9.6	180.8	11.9	176.2	14.7	170.6	18.2	163.6
225	0.7	5.5	214.0	6.9	211.2	8.6	207.8	10.8	203.4	13.4	198.2	16.6	191.8		
250	0.8	6.2	237.6	7.7	234.6	9.6	230.8	11.9	226.2	14.8	220.4	18.4	213.2		
280	0.9	6.9	266.2	8.6	262.8	10.7	258.6	13.4	253.2	16.6	246.8	20.6	238.8		
315	1.0	7.7	299.6	9.7	295.6	12.1	290.8	15.0	285.0	18.7	277.6	23.2	268.6		
355	1.1	8.7	337.6	10.9	333.2	13.6	327.8	16.9	321.2	21.1	312.8	26.1	302.8		
400	1.2	9.8	380.4	12.3	375.4	15.3	369.4	19.1	361.8	23.7	352.6	29.4	341.2		
450	1.4	11.0	428.0	13.8	422.4	17.2	415.6	21.5	407.0	26.7	396.6	33.1	383.8		
500	1.5	12.3	475.4	15.3	469.4	19.1	461.8	23.9	452.2	29.7	440.6	36.8	426.4		
560	1.7	13.7	532.6	17.2	525.6	21.4	517.2	26.7	506.6						
630	1.9	15.4	599.2	19.3	591.4	24.1	581.8	30.0	570.0						
710	2.0	17.4	675.2	21.8	666.4	27.2	655.6								
800	2.0	19.6	760.8	24.5	751.0	30.6	738.8								
900	2.0	22.0	856.0	27.6	844.8										
1000	2.0	24.5	951.0	30.6	938.8										

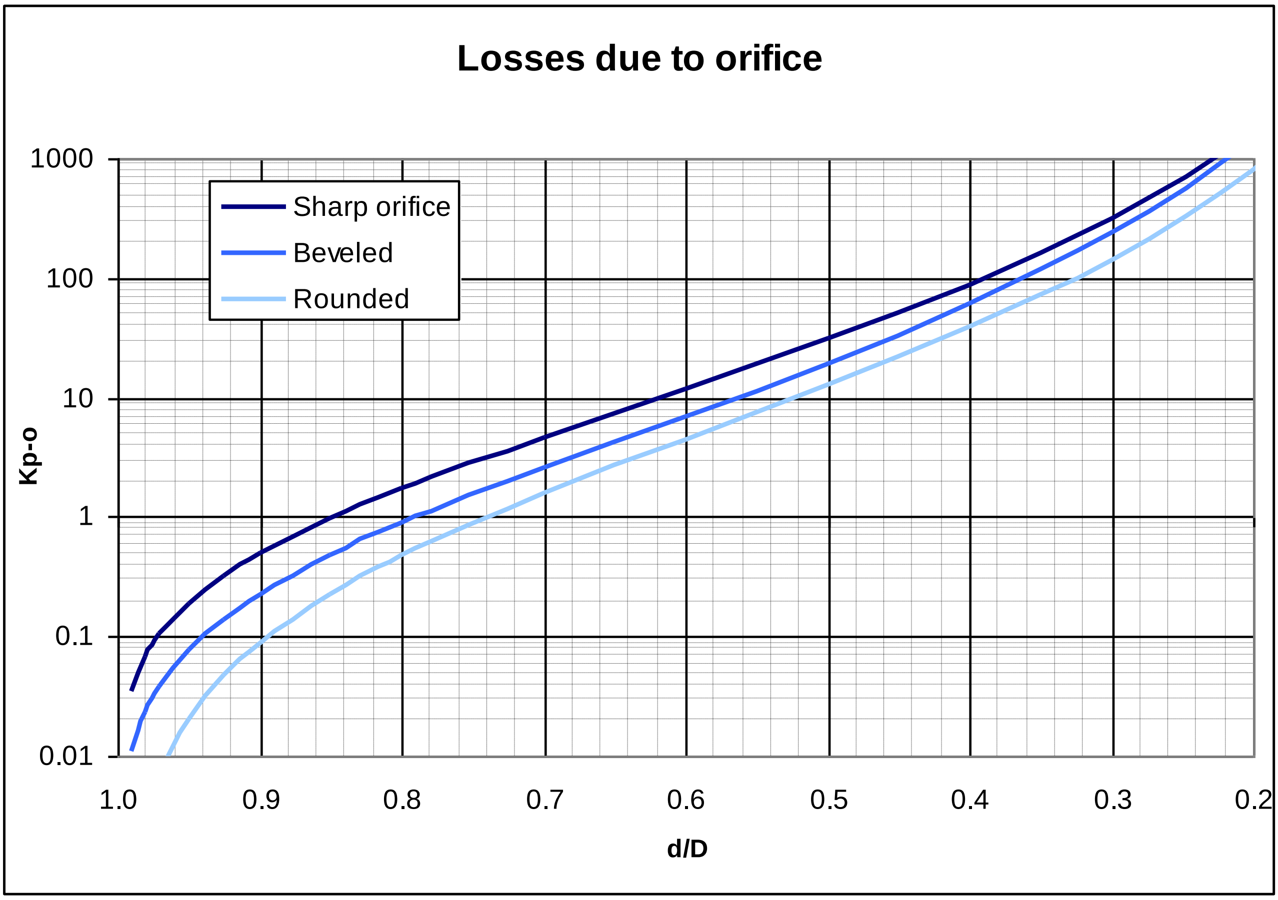
Annexe D. Diamter for cast iron pipe

DN	e	OD	Kg/m	Lining	ID
60	6	77	10.8	3.5	58.0
80	6	98	14	3.5	79.0
100	6.1	118	17.4	3.5	98.8
125	6.2	144	21.8	3.5	124.6
150	6.3	170	26.2	3.5	150.4
200	6.4	222	35.2	3.5	202.2
250	6.8	274	45.9	3.5	253.4
300	7.2	326	57.6	3.5	304.6
350	7.7	378	76.4	5	352.6
400	8.1	429	90.6	5	402.8
450	8.6	480	106.7	5	452.8
500	9	532	123.1	5	504.0
600	9.9	635	159.7	5	605.2
700	10.8	738	205.4	6	704.4
800	11.7	842	251.3	6	806.6
900	12.6	945	300.7	6	907.8
1000	13.5	1048	354.3	6	1009.0
1100	14.4	1151	412	6	1110.2
1200	15.3	1255	474.2	6	1212.4
1400	17.1	1462	641.6	9	1409.8

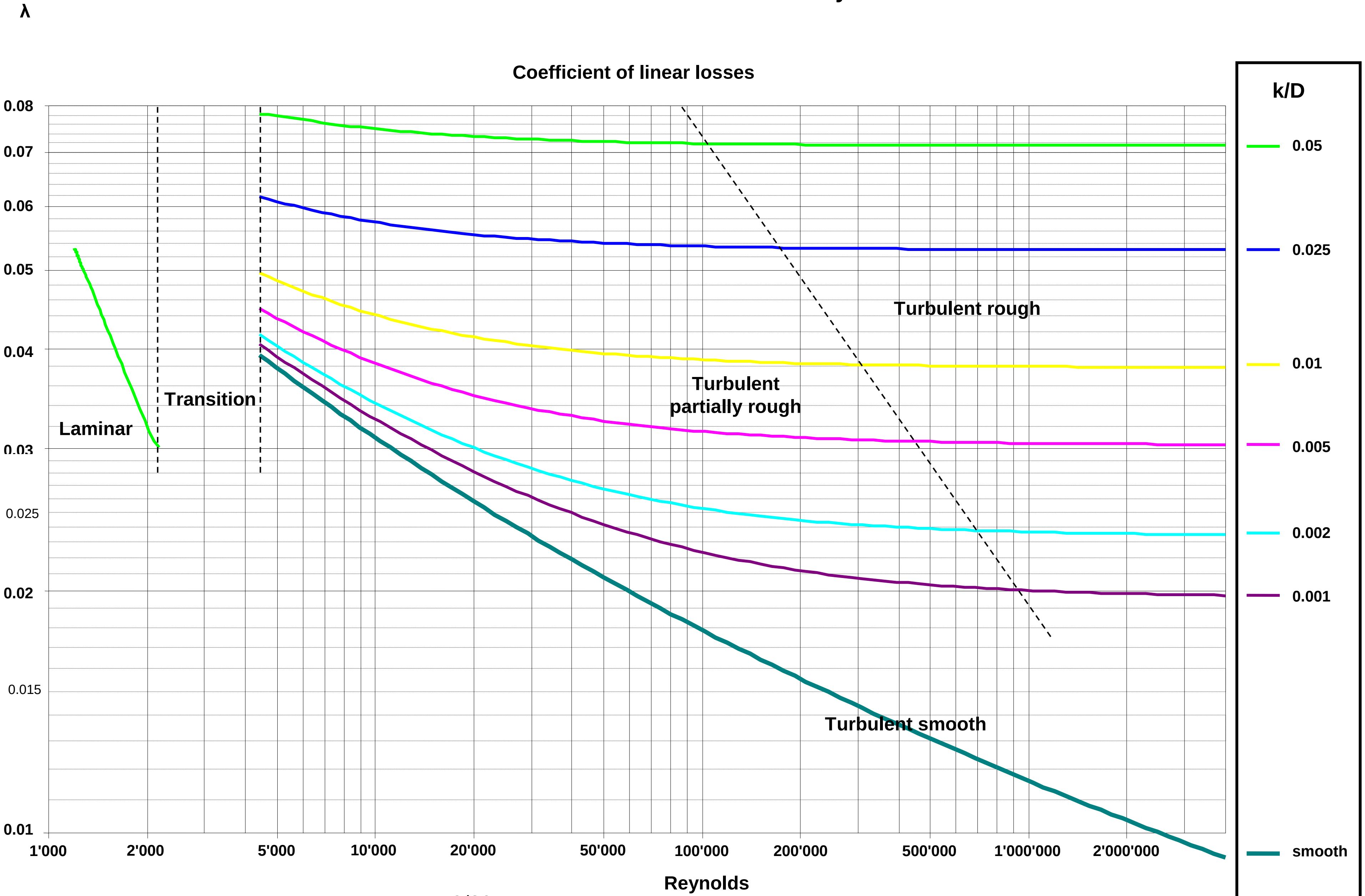
Annexe E. Punctual friction losses



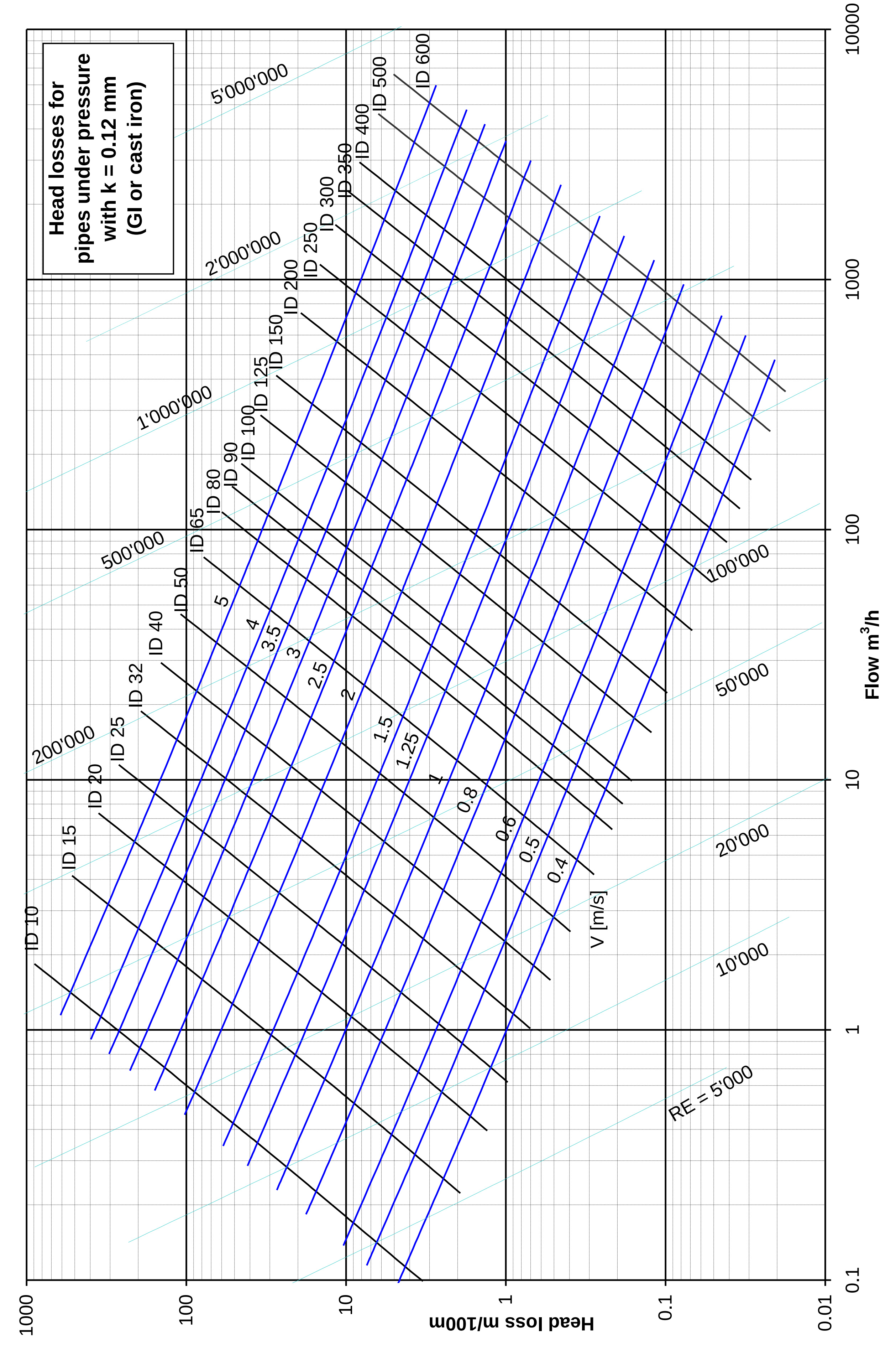


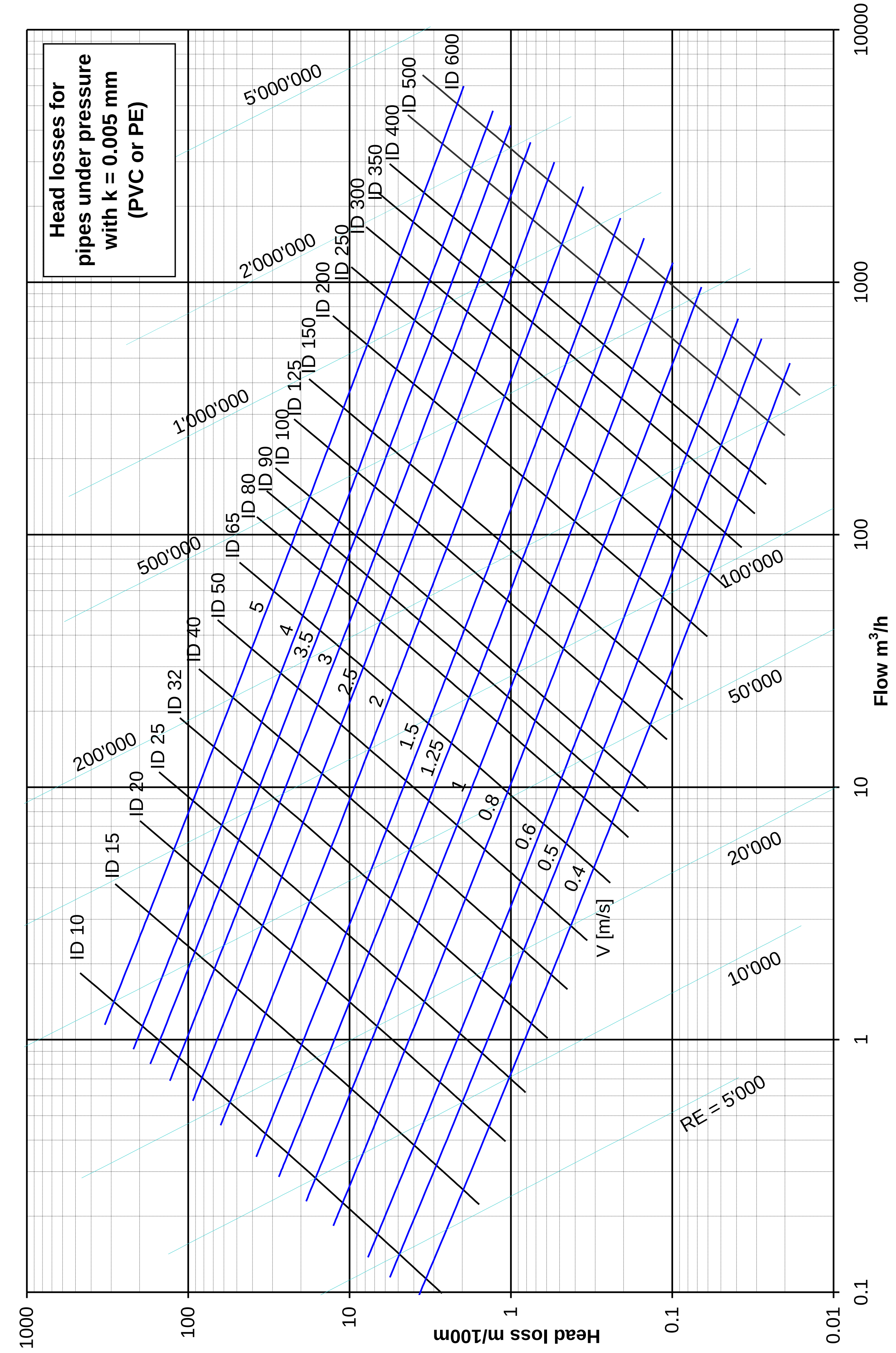

Losses due to orifice
1000
100
10
1
0.1
0.01
Kp-o
Sharp orifice
Beveled
Rounded
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
d/D

Annexe F. Moody chart

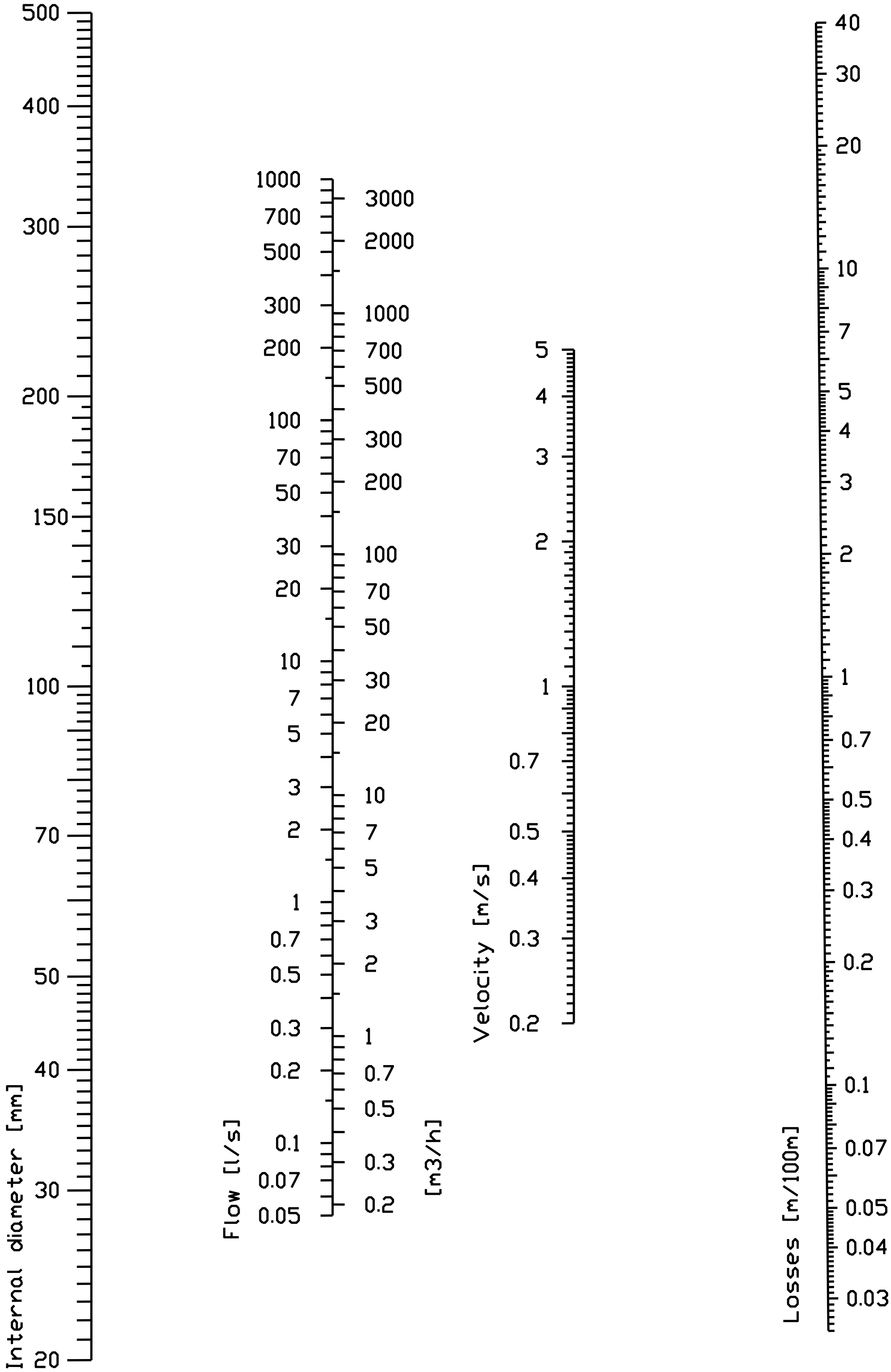


Annexe G. Charts for linear head losses

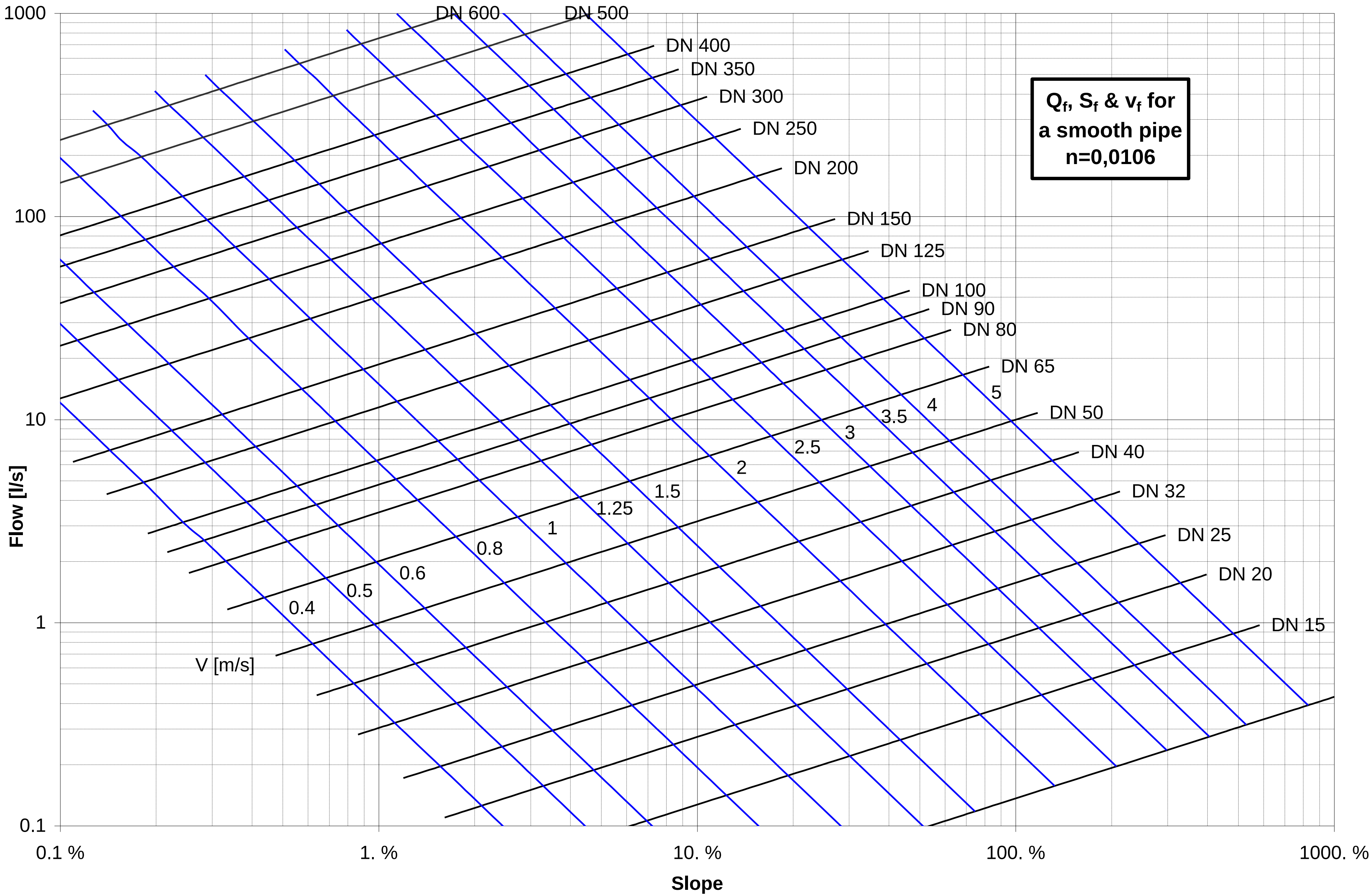


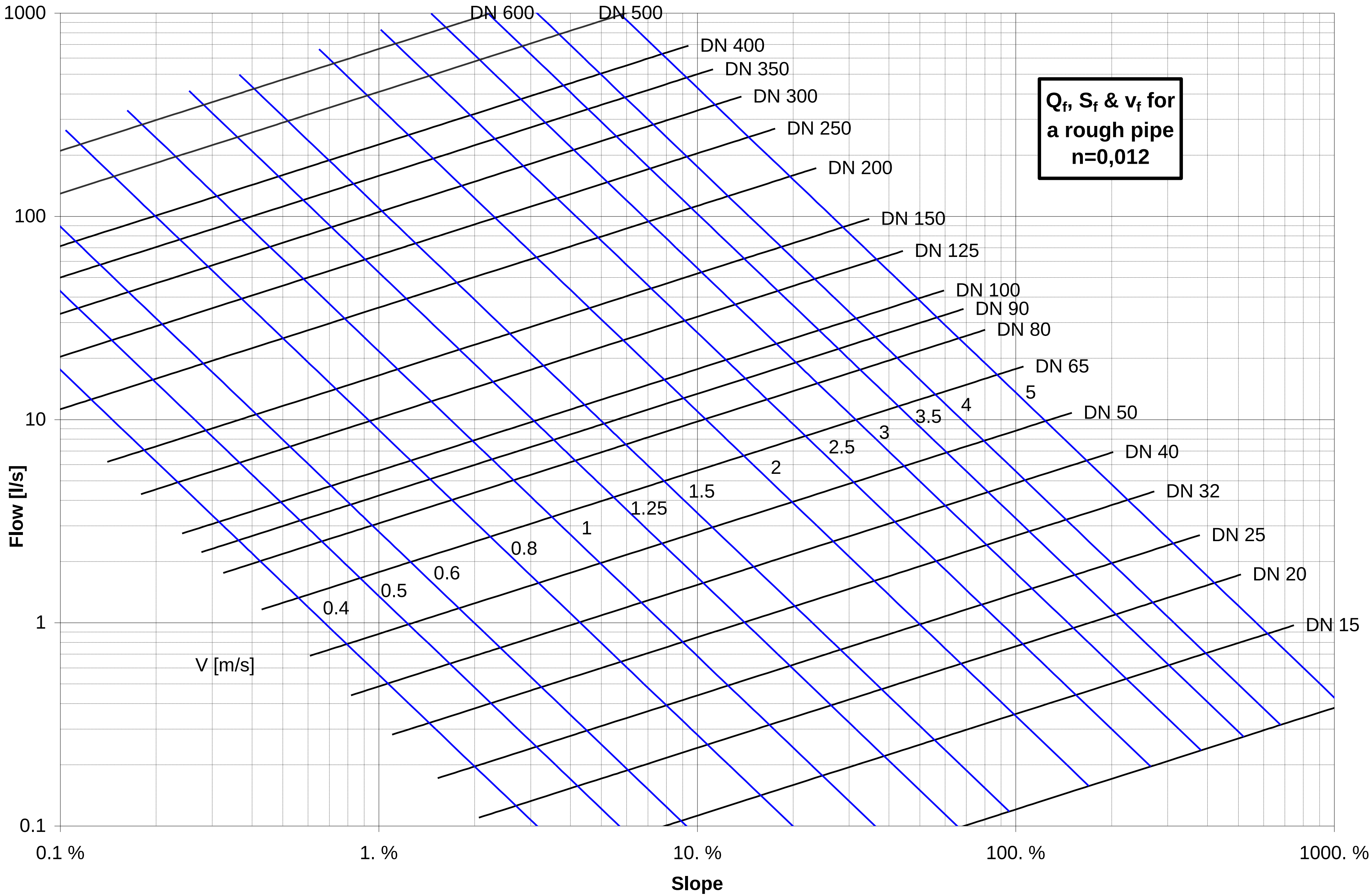

Head losses for pipes under pressure with k = 0.005 mm (PVC or PE)
Head loss m/100m
Flow m3/h
1000
100
10
1
0.1
0.01
10000
1000
100
10
1
0.1
ID 10
ID 15
ID 20
ID 25
ID 32
ID 40
ID 50
ID 65
ID 80
ID 90
ID 100
ID 125
ID 150
ID 200
ID 250
ID 300
ID 350
ID 400
ID 500
ID 600
V [m/s]
5
4
3.5
3
2.5
2
1.5
1.25
1
0.8
0.6
0.5
0.4
RE = 5'000
10'000
20'000
50'000
100'000
200'000
500'000
1'000'000
2'000'000
5'000'000

Annexe H. Log scale chart for linear friction losses



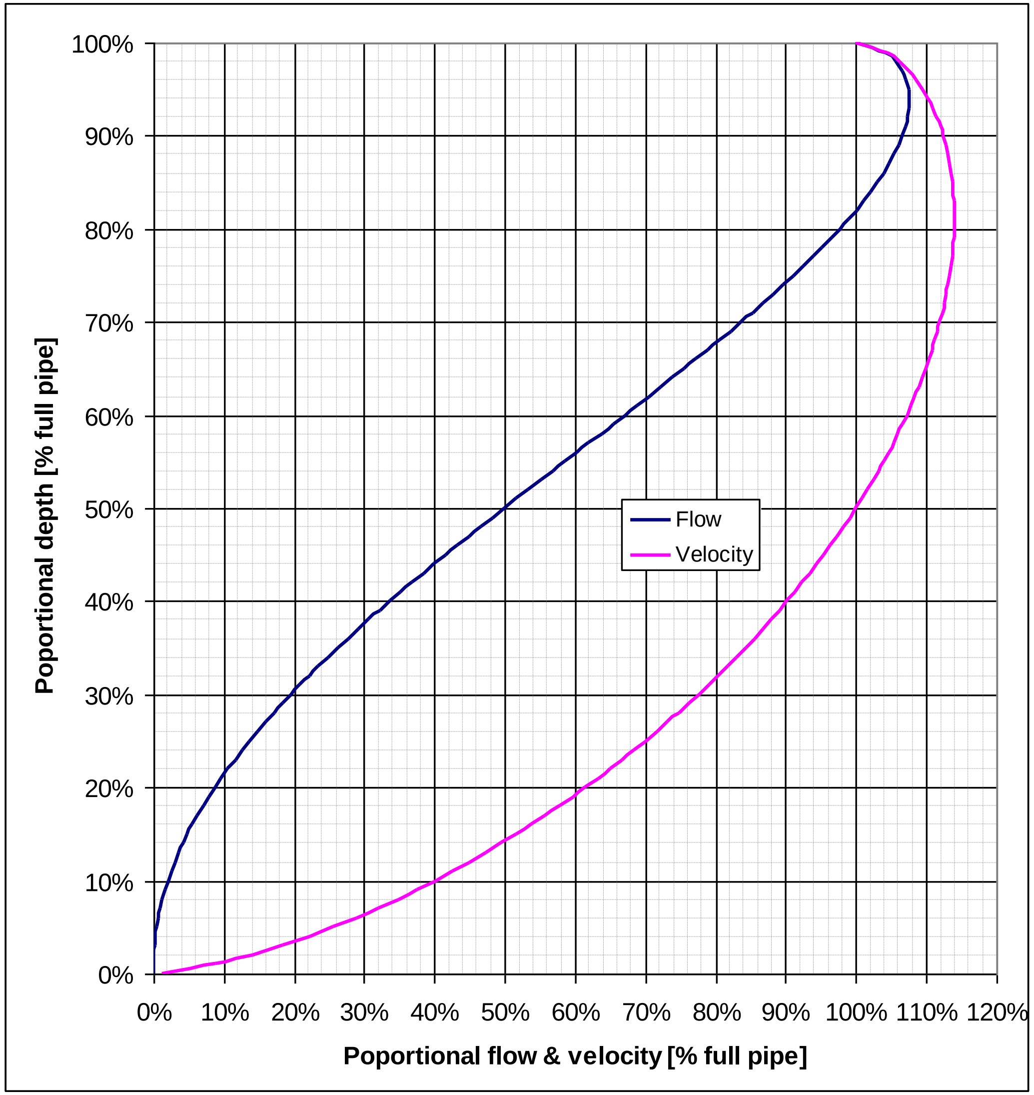
Annexe I. Gravity system, slope flow and velocity to have the pipe full



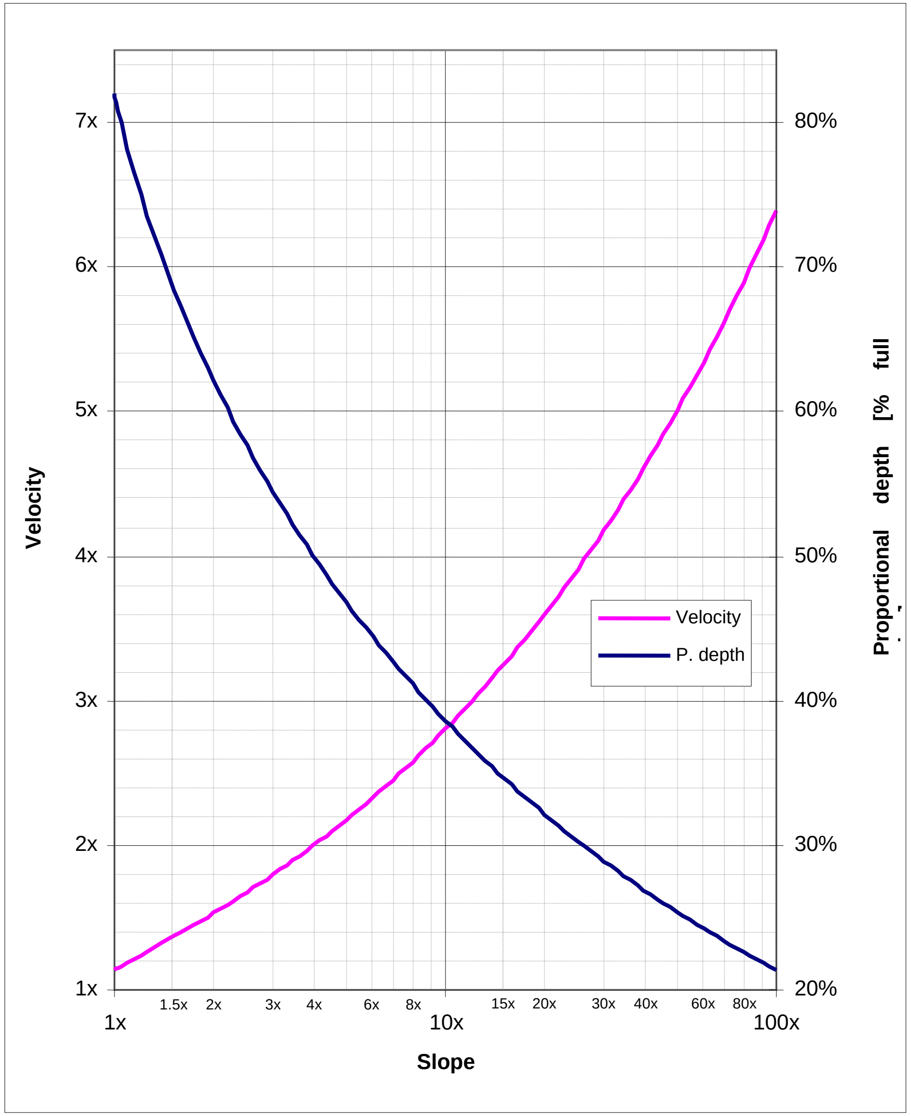

Qf, Sf & vf for a rough pipe n=0,012
Flow [l/s]
Slope
V [m/s]
DN 600
DN 500
DN 400
DN 350
DN 300
DN 250
DN 200
DN 150
DN 125
DN 100
DN 90
DN 80
DN 65
DN 50
DN 40
DN 32
DN 25
DN 20
DN 15
0.4
0.5
0.6
0.8
1
1.25
1.5
2
2.5
3
3.5
4
5
1000
100
10
1
0.1
0.1 %
1. %
10. %
100. %
1000. %

Annexe J. Gravity system, sub charts

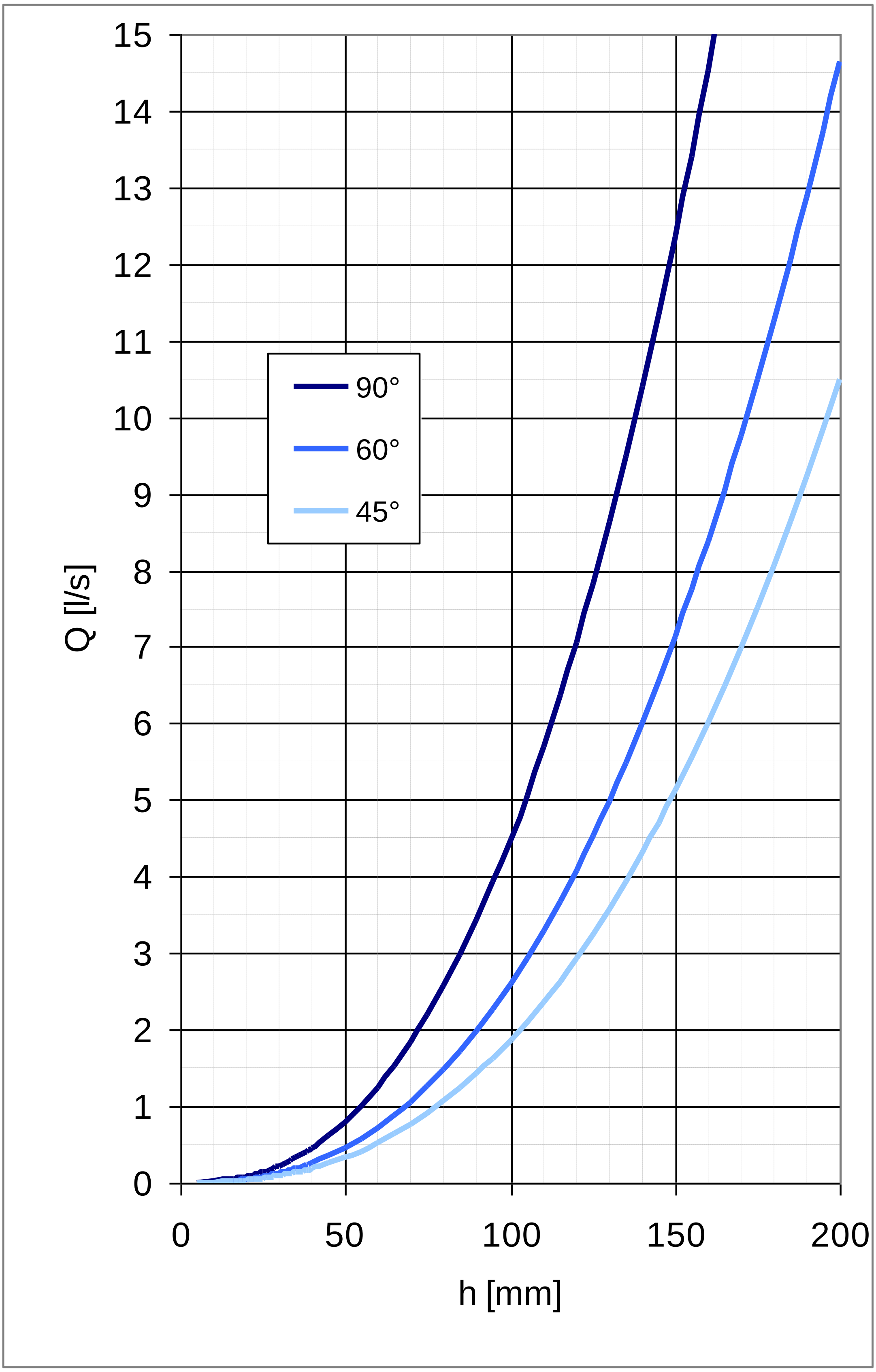
For fixed slope, depth and velocity according to a varying flow



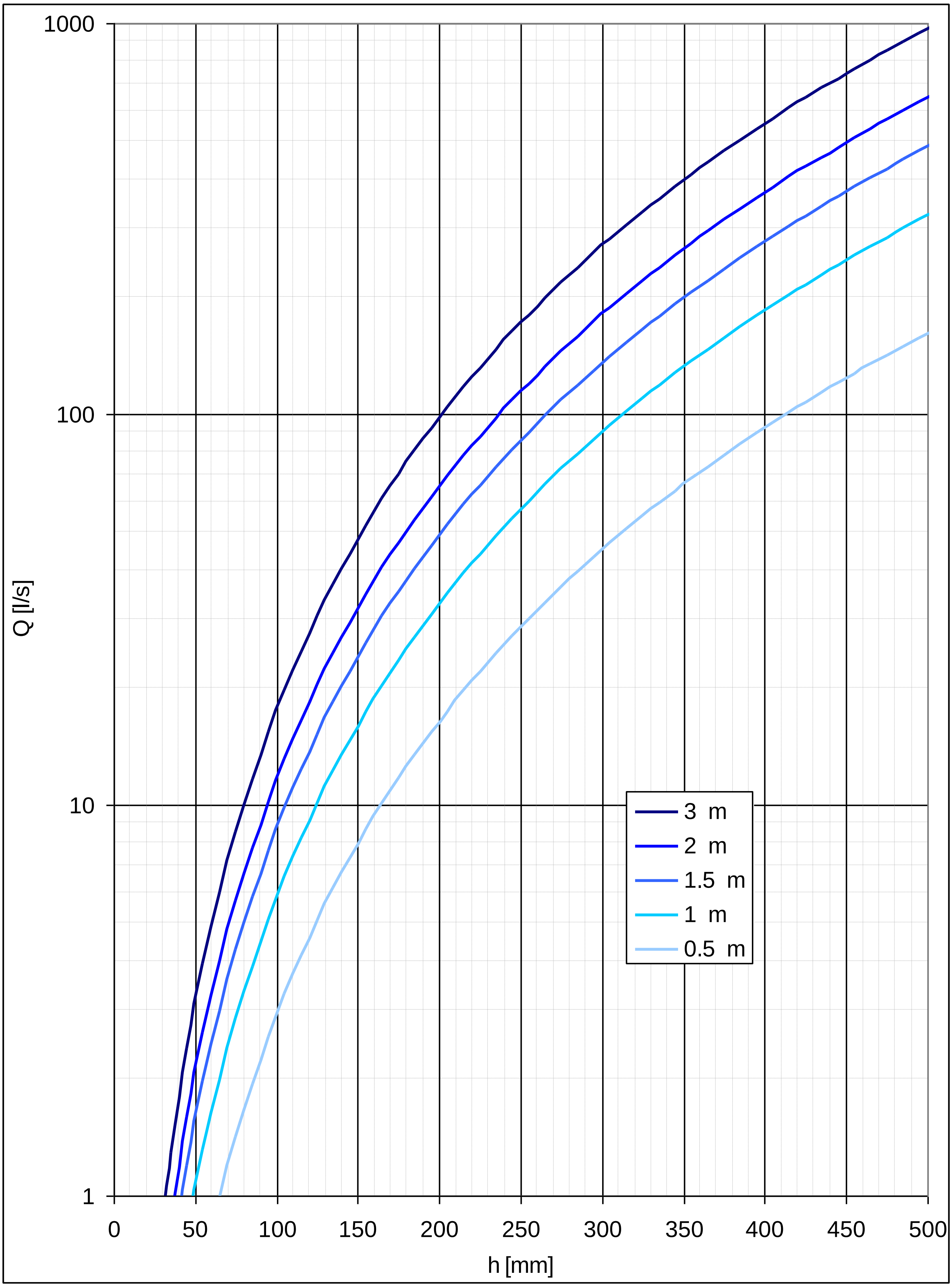
For fixed flow, velocity and depth for a varying slope



Annexe K. Flow through a V notch



Annexe L. Flow through a square weir



Annexe M. Error propagation

When using formulas to determine a variable y, knowing other variable x_1, x_2 … x_n, the error of x_1, x_2 … x_n will be propagated to y. The following formula can be used to estimate the error of y, knowing the one of x.

If we have $y = f(x_1, x_2, ..., x_n)$, then the error of x:

$$\mathrm{Error}(y) = \frac{\partial f}{\partial x_1} \cdot \Delta x_1 + \frac{\partial f}{\partial x_2} \cdot \Delta x_2 + ... + \frac{\partial f}{\partial x_n} \cdot \Delta x_n$$

Example 1:

We want to know the flow through a pipe and its error, knowing the velocity and the diameter of the pipe

v=1.5 m/s ± 5%

D=60mm ± 0.6mm

The flow would be $Q = \frac{v \cdot \pi \cdot D^2}{4} = \frac{1.5 \times \pi \times 0.06^2}{4} = 0.00424 m^3/s = 4.24 l/s$

So we have $Q = f(v, D)$

Now to assess the error of Q (variable y):

For the first variable x_1, the velocity v

The derivative of the function f with respect to the velocity: $\frac{\partial f}{\partial v} = \frac{\pi \cdot D^2}{4} = \frac{\pi \times 0.06^2}{4} = 0.0028$

$\Delta v = 1.5 \times 5\% = 1.5 \times 0.1 = 0.075$

For the second variable v_2, the diameter D

$$\frac{\partial f}{\partial D} = \frac{v \cdot \pi \cdot 2D}{4} = \frac{1.5 \times \pi \times 2 \times 0.06}{4} = 0.0028 = 0.142$$

$\Delta D = 0.0006m$

$$\mathrm{Error}(Q) = \frac{\partial f}{\partial v} \cdot \Delta v + \frac{\partial f}{\partial D} \cdot \Delta D = 0.0028 \times 0.075 + 0.142 \times 0.0006 = 0.21 + 0.0852 = 0.00029 m^3/s = 0.29 l/s$$

As it can be seen, the error of the velocity has a bigger influence on the error of the flow than the error of the diameter.

In percentage, this gives us

$$\frac{0.29 l/s}{4.42 l/s} \times 100\% = 6.83\%$$

Therefore, the flow is 4.2 l/s ± 0.3 l/s

Example 2:

What is the flow and its error through a V-notch weir, if we have

h=15cm±0.1cm

θ=45°θ±1°

The flow through a V-notch is $Q = \frac{4}{5} c \cdot tg\left(\frac{\theta}{2}\right) \cdot \sqrt{2g} \cdot h^{5/2}$

In this case $Q = \frac{4}{5} \times 0.4 \times tg\left(\frac{\pi/2}{2}\right) \times \sqrt{2 \times 9.81} \times 0.15^{5/2} = 0.012 m^3/s = 12.35 l/s$

To simplify this example, we will assume that c=0.4 and g=9.81 are constant, and they do not have errors. So, we have Q=f(θ, h)

- The first variable x_1, the angle θ

The derivative of the function f with respect to the velocity:

$$\frac{\partial f}{\partial \theta}=\frac{4}{5}c\cdot\frac{1}{2\cdot\left(\cos\left(\frac{\theta}{2}\right)\right)^2}\cdot\sqrt{2g}\cdot h^{5/2}=\frac{4}{5}\times 0.4\times\frac{1}{2\times\left(\cos\left(\frac{\pi/2}{2}\right)\right)^2}\times\sqrt{2\times 9.81}\times 0.15^{5/2}=0.012$$

Δθ = 1° = 0.017 rad

$$\frac{\partial f}{\partial \theta}\cdot\Delta\theta=0.012\times 0.017=0.0002$$

- The second variable x_2, the height h,

The derivative of the function f with respect to h is:

$$\frac{\partial f}{\partial h}=\frac{4}{5}c\cdot tg\left(\frac{\theta}{2}\right)\cdot\sqrt{2g}\cdot\frac{5}{2}h^{3/2}=\frac{4}{5}\times 0.4\times tg\left(\frac{\pi/2}{2}\right)\times\sqrt{2\times 9.81}\times\frac{5}{2}0.15^{3/2}=0.206$$

Δh=0.001 m

Therefore, $\frac{\partial f}{\partial h}\cdot\Delta h=0.206\times 0.001=0.0036$

$$\mathrm{Error(Q)}=\frac{\partial \mathrm{f}}{\partial \theta}\cdot\Delta\theta+\frac{\partial \mathrm{f}}{\partial \mathrm{h}}\cdot\Delta\mathrm{h}=0.012\times 0.017+0.206\times 0.001=0.0002+0.0036=0.0038\mathrm{m}^3/\mathrm{s}=3.8\mathrm{l/s}$$

Therefore, the flow Q is

Q=12 l/s ± 4 l/s

If the formula to calculate the variable x is too difficult to derivate, numerical derivation can be done.

Annexe N. Water hammer figures for some pipes

The following tables give water hammer figures for different pipes.
First the velocity of the pressure wave for water at 20°,
the head surge (Δh [m]) assuming a variation of initial velocity(Δv) of 1 m/s,
the return time of the wave (T_r [s]) assuming a pipe length of 1 km

PE pipes (K_{PE}=1.1 GPa)			
SDR	c [m/s]	Δh [m]	T_r [s]
6	495	50	4.0
7.4	437	45	4.6
9	386	39	5.2
11	342	35	5.8
13.6	303	31	6.6
17	269	27	7.4
21	238	24	8.4
26	212	22	9.4

PVC Pipes OD <100 (K_{Pvc}=4 GPa)				
PN	SDR	c [m/s]	Δh [m]	T_r [s]
25	9	682	70	2.9
16	13.6	559	57	3.6
12.5	17	493	50	4.1
10	21	441	45	4.5
8	26	399	41	5.0
6.3	33	362	37	5.5
6	34.4	347	35	5.8
5	41	318	32	6.3

PVC Pipes OD >100 (K_{Pvc}=4 GPa)				
PN	SDR	c [m/s]	Δh [m]	T_r [s]
25	11	609	62	3.3
20	13.6	547	56	3.7
16	17	490	50	4.1
12.5	21	440	45	4.5
10	26	393	40	5.1
8	33	352	36	5.7
6.3	41	315	32	6.4

The water hammer for plastic pipe depend mainly of the SDR of the pipe
(with exception of very small diameter as the actual thickness is bigger)

Cast iron pipes (K_{CI}=140 GPa)			
DN	c [m/s]	Δh [m]	T_r [s]
60	1'380	141	1.4
80	1'350	138	1.5
100	1'324	135	1.5
125	1'294	132	1.5
150	1'266	129	1.6
200	1'216	124	1.6
250	1'182	120	1.7
300	1'154	118	1.7
350	1'134	116	1.8
400	1'114	114	1.8
450	1'100	112	1.8
500	1'085	111	1.8
600	1'062	108	1.9
700	1'045	107	1.9
800	1'031	105	1.9
900	1'019	104	2.0
1'000	1'009	103	2.0
1'100	1'001	102	2.0
1'200	994	101	2.0
1'400	982	100	2.0

For cast iron pipes, the thickness is not changing according to the PN thus the values only depend on the diameter.

Thickness of concrete lining was neglected